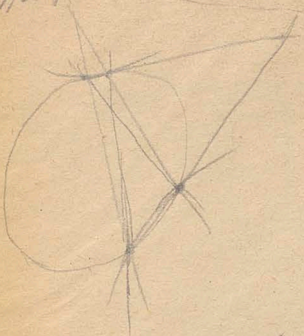
Dr. Emil Weyr IV. S. 92.

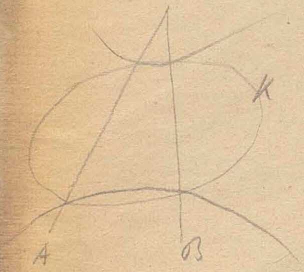
Analytische Geometrie.

M Smoluchowski

$a_{11}x^2$ [illegible]



4 [illegible]



$A=0 \quad B=0$

$AB=0$: [illegible]

$K=0$

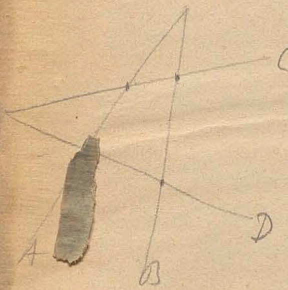
$K+\lambda AB=0$ [illegible]

[illegible] λ AB [illegible]

K [illegible]

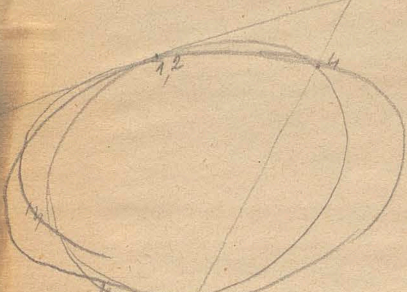
[illegible]

$AB+\lambda CD=0$

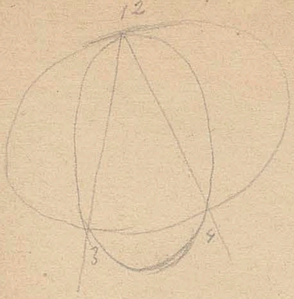


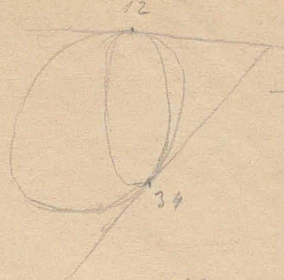
[illegible] 2 [illegible]

[illegible]; 3 — oscul.

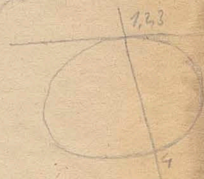


$K+\lambda AB=0$



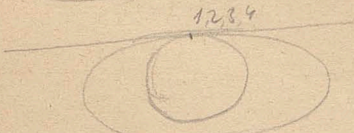


Oscul.





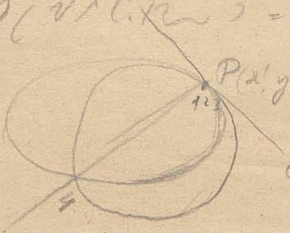
← $K + \lambda A^2 = 0$



Oscul. [illegible] 2 [illegible] 3. [illegible]

[illegible] 3 [illegible] = Oscul [illegible]

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$



$P(x', y')$ [illegible]

[illegible] P [illegible] oscul. [illegible]

$K + \lambda AB = 0$ $\quad B = m(x-x') + n(y-y') = 0$

$\frac{xx'}{a^2} + \frac{yy'}{b^2} = 1$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 + \lambda\left(\frac{xx'}{a^2} + \frac{yy'}{b^2} - 1\right)\left[m(x-x') + n(y-y')\right] = 0$

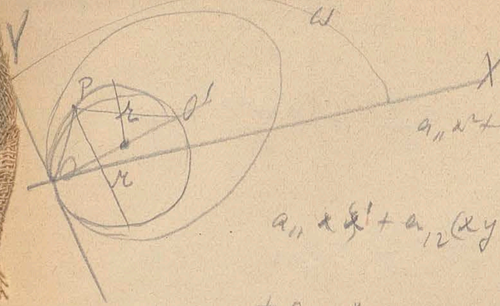
[illegible] : [illegible] —— Coeff. [illegible] x^2 = C. [illegible] y^2 [illegible]

[illegible] P [illegible]

$K + \lambda K^2 = 0$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 + \lambda\left[\frac{xx'}{a^2} + \frac{yy'}{b^2} - 1\right]^2 = 0$

m. Parabel $a_{11} a_{22} - a_{12}^2 = 0$



[illegible]

$a_{11}x^2+$

~~$a_{13}=0$~~

[illegible]

$$a_{11}xx' + a_{12}(xy'+x'y) + a_{22}yy' +$$

$$+ a_{13}x \quad\text{———}\quad = 0$$

$a_{13}x + a_{23}y = 0 \;\Big|\; t_4 \qquad a_{23}=0$

$y = 0$

$$a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_{13}x = 0$$

$OM = x \qquad OP^2 = OM' \cdot OO'$

$MP = y \qquad\qquad = x^2 + 2xy\cos\omega + y^2$

$OM' = OM\sin\omega = x\sin\omega$

~~OP^2~~ $= x^2 + 2xy\cos\omega + y^2 - 2r\sin\omega . x = 0$

$$\left.\begin{aligned} a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_{13}x = 0 \\ x^2 + 2xy\cos\omega + y^2 - 2r\sin\omega . x = 0 \end{aligned}\right\} \Big| - a_{22}$$

$$(a_{11} - a_{22})x^2 + 2(a_{12} - a_{22}\cos\omega)xy + 2(a_{13} + r a_{22}\sin\omega)x = 0$$

$$x\left[(a_{11}-a_{22})x + 2(a_{12} - a_{22}\cos\omega)y + 2(a_{13} + r a_{22}\sin\omega)\right] = 0$$

2 [illegible] $\begin{cases} x = 0 \text{ ——— } Y_{00} \\ (a_{11}-a_{22})x + 2(a_{12}-a_{22}\cos\omega)y + 2(a_{13} + r a_{22}\sin\omega) = 0 \end{cases}$

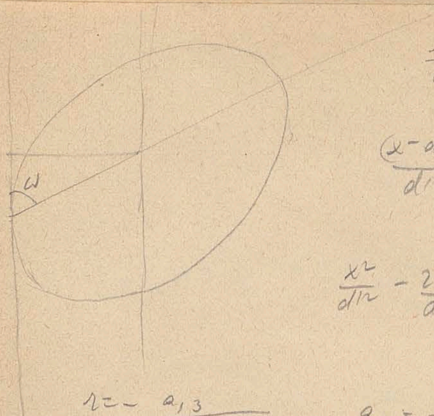
μ (34)

[illegible] O [illegible] Osc. [illegible]

$a_{13} + r a_{22}\sin\omega = 0 \;\Big|\;$ Rad. [illegible] : $r = -\dfrac{a_{13}}{a_{22}\sin\omega}$

$$\frac{x'^2}{d'^2} + \frac{y'^2}{d''^2} - 1 = 0$$

$$\frac{(x-d')^2}{d'^2} + \frac{y^2}{d''^2} - 1 = 0$$

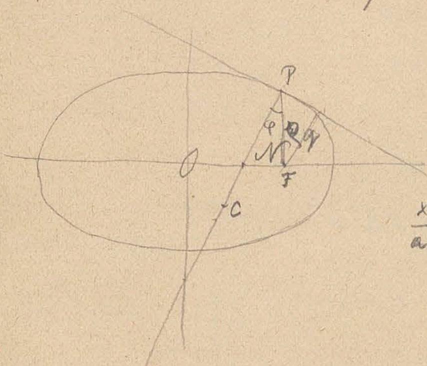


$$\frac{x^2}{d'^2} - \frac{2x}{d'} + \frac{y^2}{d''^2} = 0$$

$r = -\frac{a_{13}}{a_{22}\sin\omega}$ $\qquad a_{13} = -\frac{1}{d'}$

$$r = \frac{1}{d'\frac{\sin\omega}{d''^2}} = \frac{d''^2}{d'\sin\omega}$$

$p = d'\sin\omega$ $\qquad r = \frac{d''^2}{p}$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

C = [illegible]

$$\frac{xx'}{a^2} + \frac{yy'}{b^2} = 1$$

$-\frac{x'b^2}{y'a^2}$ $\qquad \frac{y'a^2}{x'b^2}$ [illegible]

$$y - y' = \frac{y'a^2}{x'b^2}(x - x') \quad \Big| \quad y = 0$$

$$x = -\frac{x'b^2}{a^2} + x' = ON = x'\frac{a^2-b^2}{a^2}$$

$$PN^2 = \left[x' - x'\frac{a^2-b^2}{a^2}\right]^2 + y'^2 = [illegible] = x'^2\frac{b^4}{a^4} + y'^2 = \frac{b^2}{a^2}\left[\frac{b^2x'^2}{a^2} + \frac{a^2y'^2}{b^2}\right]$$

$$= d''^2$$

$$PN = \frac{bd''}{a}$$

$\rho = a - e x'$ $\qquad \cos\varphi = \frac{q}{\rho}$

$$-\frac{\frac{xx'}{a^2} + \frac{yy'}{b^2} - 1}{\sqrt{\frac{x'^2}{a^4} + \frac{y'^2}{b^4}}} \Bigg|_{\substack{x=c \\ y=0}} = q \qquad q = 1 - \frac{c x'}{a^2}$$

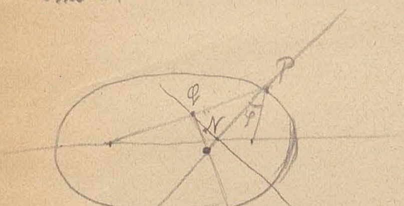
$$q = \frac{ab\left(1 - \frac{e x'}{a}\right)}{d''} = \frac{b(a - e x')}{d''} = \frac{b\rho}{d''}$$

$$\cos\varphi = \frac{b}{d''}$$

$$p = \frac{1}{\sqrt{\frac{x'^2}{a^4} + \frac{y'^2}{b^4}}} = \frac{ab}{\sqrt{\frac{b^2 x'^2}{a^2} + \frac{a^2 y'^2}{b^2}}} = \frac{ab}{d''}$$

$$r = \frac{d''^2}{\frac{ab}{d''}} = \frac{d''^3}{ab} = \frac{PN}{\cos^2\varphi}$$

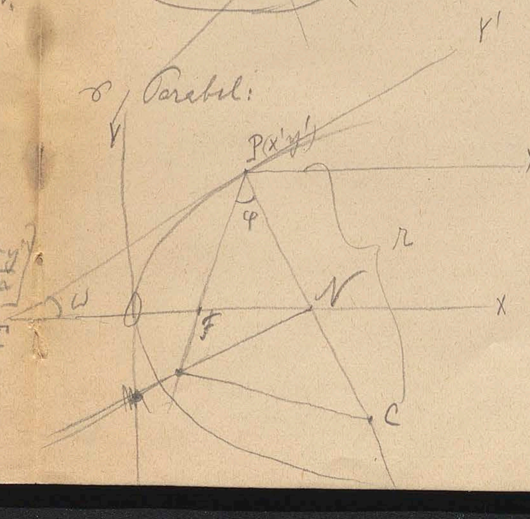
Constr.



$NQ \perp NP$

$QC \perp QP$

γ) Parabel:



$y^2 = 2p'x$

$y^2 - 2p'x = 0 \qquad p' = \frac{p}{\sin^2\omega}$

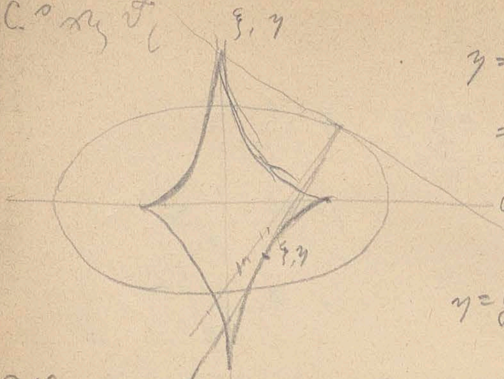
$$r = -\frac{a_{13}}{a_{22}} \sin\omega$$

$a_{22} = 1 \qquad a_{13} = -p'$

$$r = \frac{p'}{\sin\omega} = \frac{p}{\sin^3\omega}$$

$TN = p' \quad = \frac{PN}{\cos^2\varphi}$

$C^o\ xy\ \vartheta_i$



$$\eta = y' - PE \cos\varphi$$

$$= y' - r\cos\varphi$$

$$\cos\varphi = \frac{y'}{PN}$$

$$\eta = y' - \frac{d''^2}{p}\frac{y'}{PN}$$

PN —— $\frac{b d''}{a}$

$$\eta = y'\left[1 - \frac{d''^2}{p}\frac{a}{b d''}\right]$$

$$= y'\left[1 - \frac{a d''}{p b}\right] = y'\left[1 - \frac{a d''^2}{a b^2}\right]$$

$$d''^2 = \frac{b^2 y'^2}{a^2}$$

$$= y'\frac{b^2 - d''^2}{b^2} = y'\frac{b^2 - \left(\frac{b^2 x'^2}{a^2} - \frac{a^2 y'^2}{b^2}\right)}{b^2}$$

$$= y'\left[\quad\right] = y'\frac{(3b^2 - a^2)}{b^4}$$

Evolute in dim.

$$x' = \sqrt[3]{\frac{a^4 \xi}{c^2}} \qquad y' = -\sqrt[3]{\frac{b^4 \eta}{c^2}}$$

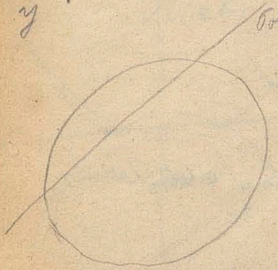
$$\frac{1}{a^2}\sqrt[3]{\frac{a^8 \xi^2}{c^4}} + \frac{1}{b^2}\sqrt[3]{\frac{b^8 \eta^2}{c^4}} = 1$$

$$\sqrt[3]{\frac{a^2 \xi^2}{c^4}} + \sqrt[3]{\frac{b^2 \eta^2}{c^4}} = 1$$

$$a^{\frac{2}{3}} \xi^{\frac{2}{3}} + b^{\frac{2}{3}} \eta^{\frac{2}{3}} = c^{\frac{4}{3}}$$

$$a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + 2a_{13}x_1x_3 + 2a_{23}x_2x_3 = 0$$

y



$$y_1(a_{11}x_1 + a_{12}x_2 + a_{13}x_3) +$$
$$+ y_2(a_{21}x_1 + a_{22}x_2 + a_{23}x_3) +$$
$$+ y_3(a_{31}x_1 + a_{32}x_2 + a_{33}x_3) = 0$$

$$a_{ik} = a_{ki}$$

[illegible]

$$x_1(a_{11}y_1 + a_{12}y_2 + a_{13}y_3) + x_2(a_{21}y_1 + a_{22}y_2 + a_{23}\ \ \ ) \dots$$

$$+ x_3(\qquad\qquad) = 0$$

$$\rho\,\eta_1 = a_{11}y_1 + a_{12}y_2 + a_{13}y_3$$
$$\rho\,\eta_2 = a_{21}y_1 + a_{22}y_2 + a_{23}y_3$$
$$\rho\,\eta_3 = a_{31}y_1 + a_{32}y_2 + a_{33}y_3$$

} [illegible] C. o C. [illegible] Polare

σ / η.

$$y_1\eta_1 + y_2\eta_2 + y_3\eta_3 = 0$$

[illegible]

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \text{Discrim.}$$

$$\sigma y_1 = A_{11}\eta_1 + A_{21}\eta_2 + A_{31}\eta_3$$
$$\sigma y_2 = A_{12}\eta_1 + A_{22}\eta_2 + A_{32}\eta_3$$
$$\sigma y_3 = A_{13}\eta_1 + A_{23}\eta_2 + A_{33}\eta_3$$

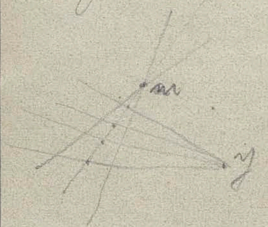
} η_1 C. o C. [illegible] C. e C. η_2 η_3

[illegible] C. [illegible] C.

$$0 = A_{11}\eta_1^2 + A_{22}\eta_2^2 + A_{33}\eta_3^2 + 2A_{12}\eta_1\eta_2 + \cdots$$

$=$ [illegible] C. [illegible] C_0.

[illegible] Discr. $=$ [illegible]



[illegible]

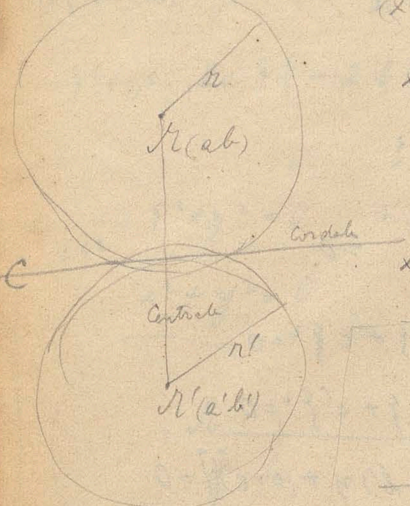
$$y_1\,(a_{11}m_1 + a_{12}m_2 + a_{13}m_3)$$
$$+\,y_2\,(\qquad\qquad\qquad\qquad)$$
$$+\,y_3\,(\qquad\qquad\qquad\qquad) = 0$$ [illegible] y

[illegible] Coeff. $= 0$ [illegible]

$$\left.\begin{array}{l} a_{11}m_1 + a_{12}m_2 + a_{13}m_3 = 0 \\ a_{21} \\ a_{31}m_1 + \qquad\qquad a_{33}m_3 = 0 \end{array}\right\}$$ [illegible] Δ [illegible]

[illegible] C. [illegible]

[illegible] $\delta = 0$ [illegible] C. [illegible] 2 [illegible]



$(x-a)^2+(y-b)^2-r^2=0$

$x^2+y^2-2ax-2by+a^2+b^2-r^2=0$

$x^2+y^2-2a'x-2b'y+a'^2+b'^2-r'^2=0$

[illegible] $K=0$ [illegible] Curve
$K'=0$ [illegible]

[illegible] $K-\lambda K'=0$ [illegible] 3 Curve [illegible]
[illegible]

[illegible] C. K [illegible] $K'=0$ [illegible] $K-\lambda K'=0$.

$\lambda=1$ $K-K'=0$ [illegible] C.

[illegible]:

$2(a-a')x+2(b-b')y+a'^2+b'^2-r'^2-a^2-b^2+r^2=0$

[illegible] 3ten [illegible] = [illegible] = Cordale

[illegible] Centrale [illegible] $\perp$ [illegible] Const. = − rec. [illegible]

$-\left(\frac{a-a'}{b-b'}\right)$ $\left(\frac{b-b'}{a-a'}\right)$

[illegible]

$a^2+b^2-r^2=c$

$x^2+y^2-2ax-2by+c=0$

$x=\frac{a_1x_1+a_2x_2+a_3x_3}{c_1x_1+c_2x_2+c_3x_3}=\frac{\xi}{\zeta}$ $y=\frac{b_1x_1+b_2x_2+b_3x_3}{c_1x_1+c_2x_2+c_3x_3}=\frac{\eta}{\zeta}$

$c_1 x_1 + c_2 x_2 + c_3 x_3 = \lambda \zeta$ [illegible]

[illegible] ∞

~~$x = \frac{\xi}{\zeta}$~~ ~~$y = \frac{\eta}{\zeta}$~~

$$\frac{\xi^2}{\zeta^2} + \frac{\eta^2}{\zeta^2} - 2a \cdot \quad = 0$$

$$\xi^2 + \eta^2 - 2a\,\xi\zeta - 2b\,\eta\zeta + c\,\zeta^2 = 0$$

$$\xi^2 + \eta^2 - 2a'\,\xi\zeta - 2b'\,\eta\zeta + c'\,\zeta^2 = 0$$

$$\zeta\left[2(a-a')\,\xi + 2(b-b')\,\eta + (c-c')\,\zeta\right] = 0$$

$$\zeta = 0$$

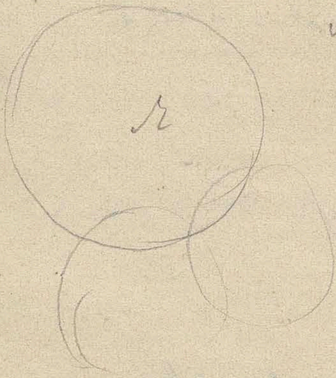
$$2(a-a')\,\xi + 2(b-b')\,\eta + (c-c')\,\zeta = 0.$$

[illegible]

[illegible]

[illegible]:

[illegible] variabel



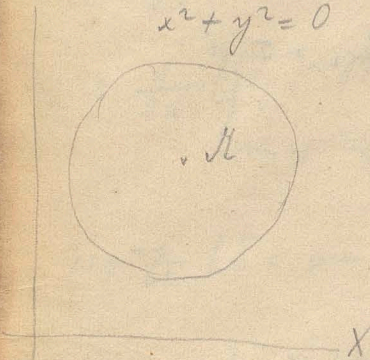
[illegible] Coord. [illegible]

$$\left.\begin{aligned}\xi^2+\eta^2-2a\,\xi\zeta-2b\,\eta\zeta+c\,\zeta^2&=0\\ \zeta&=0\end{aligned}\right\}$$

$\xi^2+\eta^2=0$ [illegible] Coord. [illegible] im. ∞ [illegible]

$:\xi^2$

$x^2+y^2=0$ [illegible] imag. [illegible]



$\frac{y}{x}=tg\,\alpha$

$1+\left(\frac{y}{x}\right)^2=0$

$1+tg^2\alpha=0$

$tg\,\alpha=\pm i=\pm\sqrt{-1}$

[illegible] α [illegible] ∞ [illegible] imag. [illegible]

$\alpha=\operatorname{arctg}(\pm i)$

$\operatorname{arctg}\alpha=\alpha-\frac{\alpha^3}{3}+\frac{\alpha^5}{5}-\frac{\alpha^7}{7}+\cdots$

$\operatorname{arctg}(\pm i)=\pm i\pm\frac{i}{3}\pm\frac{i}{5}\pm\frac{i}{7}\pm\cdots$

[illegible]

$=\pm i\left[1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\cdots\infty\right]$

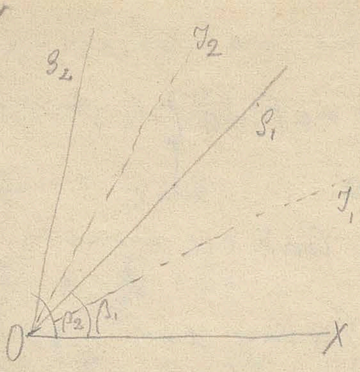
$=\pm i\infty$ divergent [illegible]

$\alpha=\pm i\infty$

[illegible] ∞ [illegible]

[illegible] metrischen [illegible]

[illegible] Recipr. [illegible]



O [illegible] imag.

e_6 [illegible]:

$J_1 OX = \alpha_1 \qquad J_2 OX = \alpha_2$

$S_1 O S_2 = \omega$

$\operatorname{tg} \alpha_1 = \pm i$

$\operatorname{tg} \alpha_2 = \mp i$

$\operatorname{tg} \alpha_1 = + i$

$\operatorname{tg}_2 = - i$

$\delta = (S_1\, S_2\, J_1\, J_2) \qquad \omega = \beta_1 - \beta_2$

$$= \frac{\sin S_1 J_1}{\sin S_2 J_1} : \frac{\sin S_1 J_2}{\sin S_2 J_2} = \frac{\sin(\beta_1 - \alpha_1)}{\sin(\beta_2 - \alpha_1)} : \frac{\sin(\beta_1 - \alpha_2)}{\sin(\beta_2 - \alpha_2)}$$

$$\delta = \frac{\sin\beta_1 \cos\alpha_1 - \sin\alpha_1 \cos\beta_1}{\sin\beta_2 \cos\alpha_1 - \sin\alpha_1 \cos\beta_2} : \frac{\sin\beta_1 \cos\alpha_2 - \sin\alpha_2 \cos\beta_1}{\sin\beta_2 \cos\alpha_2 - \sin\alpha_2 \cos\beta_2}$$

$$= \frac{\operatorname{tg}\alpha_1 - \operatorname{tg}\beta_1}{}$$

$$= \frac{\operatorname{tg}\beta_1 - i}{\operatorname{tg}\beta_2 + i} : \frac{\operatorname{tg}\beta_1 + i}{\operatorname{tg}\beta_2 + i}$$

$$= \frac{\operatorname{tg}\beta_1 \operatorname{tg}\beta_2 + i[\operatorname{tg}\beta_1 - \operatorname{tg}\beta_2] + 1}{\operatorname{tg}\beta_1 \operatorname{tg}\beta_2 + i[\operatorname{tg}\beta_1 - \operatorname{tg}\beta_2] + 1}$$

$$= \frac{1 + i \operatorname{tg}(\beta_1 - \beta_2)}{1 - i \operatorname{tg}(\beta_1 - \beta_2)} = \frac{1 + i \operatorname{tg}\omega}{1 - i \operatorname{tg}\omega}$$

$$\delta = \frac{1 + i\frac{\sin\omega}{\cos\omega}}{1 - i\frac{\sin\omega}{\cos\omega}} = \frac{\cos\omega + i\sin\omega}{\cos\omega - i\sin\omega} = \frac{e^{i\omega}}{e^{-i\omega}} = e^{2i\omega}$$

$$l\delta = 2i\omega$$

$$\omega = \frac{1}{2i}\, l\delta$$

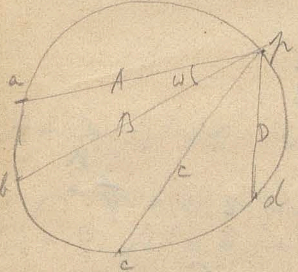
$$= \frac{1}{2i}\left\{ \text{[illegible]} \right\}$$

mag. [illegible]

$\omega = \frac{-i}{2}\, l\delta =$ [illegible] $\omega = \frac{i}{2}\, l\delta$ [illegible]

[illegible]

$\omega = \pm \frac{i}{2}\, l\delta$ [illegible]

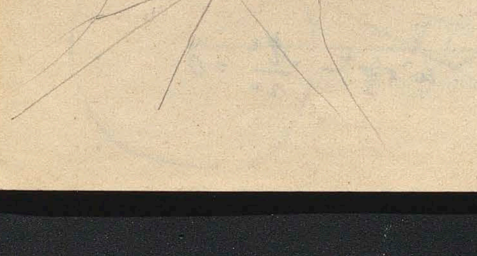


$(ABCD) = \text{const.}$, [illegible] p

[illegible]

[illegible] const. [illegible] const.

[illegible] $\measuredangle\,\omega$.



[illegible]

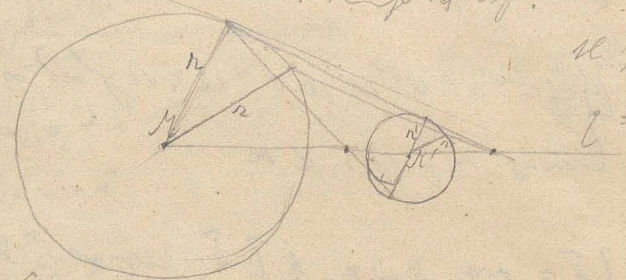
[illegible] Asympt.

[illegible]

2 [illegible] D D'

$(AA'DD') = -1$ | [illegible]

u, v = [illegible]



[illegible]

$l = r$ [illegible]

[illegible]

$$ux + vy + 1 = 0$$

$$-\frac{ux+vy+1}{\sqrt{u^2+v^2}} = 0$$ [illegible] N.

$$\frac{ux+vy+1}{\sqrt{u^2+v^2}} =$$ [illegible] $\begin{cases} x \\ y \end{cases}$

$$\frac{au+by+1}{\sqrt{u^2+v^2}} = r$$ [illegible]

[illegible]

$$(au+bv+1)^2 = r^2(u^2+v^2)$$

$au+bv+1=0$ [illegible]

$\equiv M$

$$\frac{M^2}{r^2} = u^2+v^2 \qquad u^2+v^2 - \frac{M^2}{r^2} = 0$$

$u^2+v^2-\frac{M^2}{r^2}=0$ [illegible] C.

$u^2+v^2-\frac{M'^2}{r'^2}=0$ — [illegible] —

[illegible] K, K' [illegible] C. [illegible] C.

$K-\lambda K' =$ [illegible]

$\lambda=1$

$\frac{M^2}{r^2}-\frac{M'^2}{r'^2}=0$ [illegible]

$(Mr'-M'r)(Mr'+M'r)=0$

$\frac{M}{r}-\frac{M'}{r'}=0$ } [illegible] 2 [illegible] Centrale [illegible]

$\frac{M}{r}+\frac{M'}{r'}=0$ } [M, M' [illegible]

$M-\lambda M'=$ [illegible]]

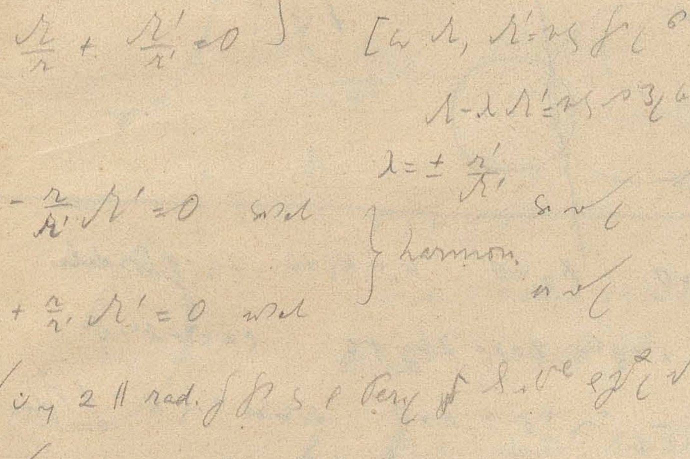
$\lambda=\pm\frac{r}{r'}$ [illegible]

$M-\frac{r}{r'}M'=0$ [illegible] } harmon. [illegible]

$M+\frac{r}{r'}M'=0$ [illegible]

[illegible] 2 ‖ rad. [illegible] Centr.

= [illegible]



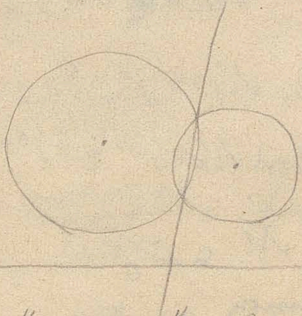
$au + bv + 1 = 0$ a, b, Coord. ...

$$A = \frac{au+bv+1}{r} - \frac{a'u+b'v+1}{r'} = 0$$

$$u\left(\frac{a}{r} - \frac{a'}{r'}\right) + v\left(\frac{b}{r} - \frac{b'}{r'}\right) + \left(\frac{1}{r} - \frac{1}{r'}\right) = 0$$

$$u \underbrace{\frac{\frac{a}{r} - \frac{a'}{r'}}{\frac{1}{r} - \frac{1}{r'}}} + v \underbrace{\frac{\frac{b}{r} - \frac{b'}{r'}}{\frac{1}{r} - \frac{1}{r'}}} + 1 = 0$$

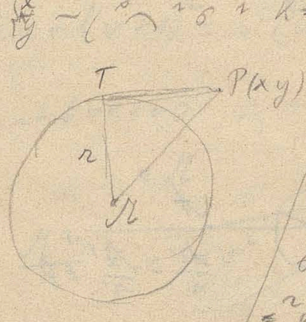
$$x = \frac{\frac{a}{r} \mp \frac{a'}{r'}}{\frac{1}{r} \mp \frac{1}{r'}}, \quad y = \frac{\frac{b}{r} \mp \frac{b'}{r'}}{\frac{1}{r} \mp \frac{1}{r'}}$$ } aussen / inner



5/5 $K_1 = 0$ $K_2 = 0$ $K_1 - K_2 = 0$ ... Cordale

$K = x^2 + y^2 - 2ax - 2by + c$ $c = a^2 + b^2 - r^2$

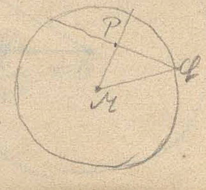
... $[\ldots]^2$



$MP^2 - r^2 = PT^2$

$K = PT^2 =$ Potenz ...

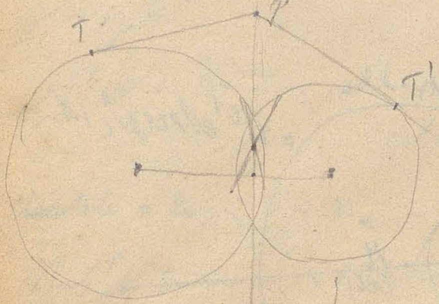
... $PQ^2 =$ Potenz

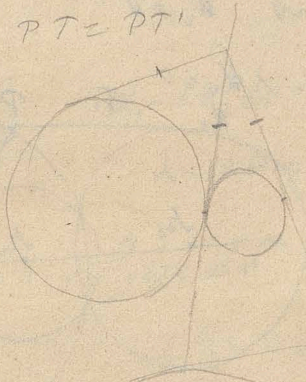


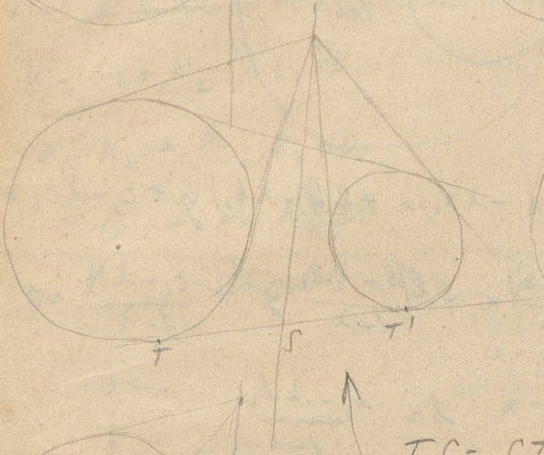
$K_1 = K_2$

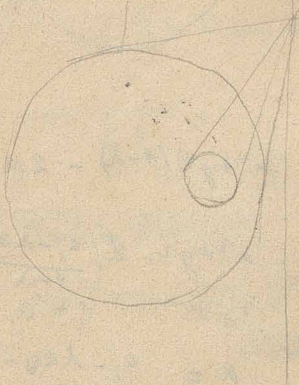
[illegible] Cordale [illegible] Potenze [illegible]

ez Cord. [illegible] Pot.





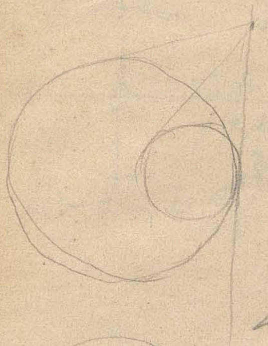


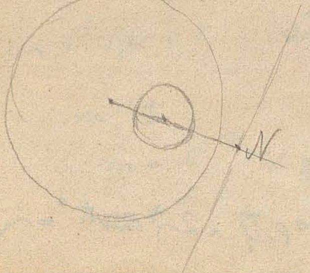


TS = ST'

[illegible] Cordale [illegible] 2 [illegible]

[illegible]





$b=0$ $b'=0$ Centri su X [illegible]

~~$x^2+y^2-2by+b^2-r^2=0$~~

$x^2+y^2-2ax+a^2-r^2=0$

$x^2+y^2-2a'x+a'^2-r'^2=0$

$2(a-a')x+a'^2-a^2+r^2-r'^2=0$

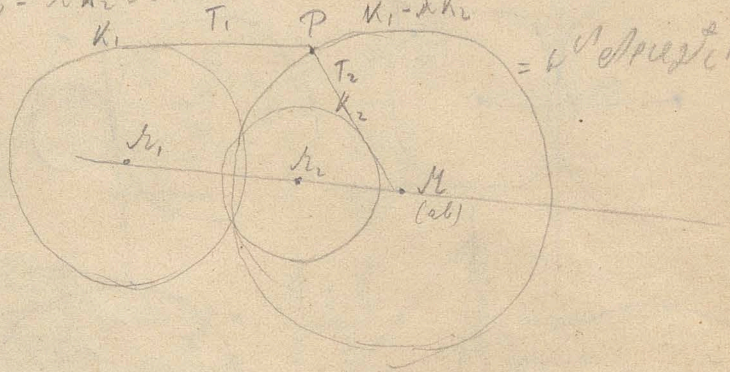
$x = \frac{r'^2-r^2+a^2-a'^2}{2(a-a')}$ = [illegible] Cord.

[illegible] $f\,x=0$

[illegible] $a^2-a'^2=r^2-r'^2$

$K_1=0 \quad K_2=0$

$K_1-\lambda K_2=0$



= [illegible]

$$(x^2+y^2)(1-\lambda)-2(a_1-\lambda a_2)x+2(b_1-\lambda b_2)y+c_1-\lambda c_2=0$$

$$x^2+y^2-2\left(\frac{a_1-\lambda a_2}{1-\lambda}\right)x+2\left(\frac{b_1-\lambda b_2}{1-\lambda}\right)y+\frac{c_1-\lambda c_2}{1-\lambda}=0$$

$$a=\frac{a_1-\lambda a_2}{1-\lambda} \qquad b=\frac{b_1-\lambda b_2}{1-\lambda}$$

$$\lambda=\frac{r_1 r}{r_2 r}$$ = [illegible] $r_1\, r_2$

$\frac{K_1}{K_2}=\lambda$ [illegible]

[illegible] cond. [illegible]

$$\frac{(PT_1)^2}{(PT_2)^2}=\lambda \qquad \frac{PT_1}{PT_2}=\sqrt{\lambda}$$

[illegible] cont. [illegible]

[illegible]

[illegible]

[illegible]

[illegible] $K_1 = 0$ $\quad K_2 = 0$ $\qquad K_1 - \lambda K_2 = 0$

$\lambda <0$ [illegible] ∞

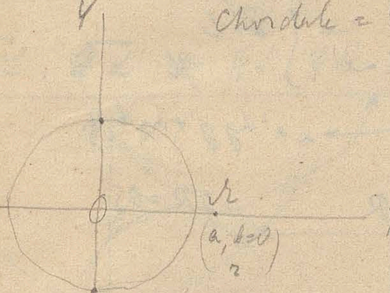
Chordale = [illegible]

[illegible] Chordale [illegible]

$$K_1 - K_2 + K_2 - \lambda K_2 = 0$$

$$K_1 - K_2 + (1-\lambda) K_2 = 0$$

$$C + \lambda' K_2^{*} = 0$$



Chordale = Y-[illegible], Centrale = X-[illegible]

$(x-a)^2 + y^2 - r^2 = 0$, $x = 0$ } [illegible] Chord.

$(x-a)^2 + y^2 - r^2 - \lambda x = 0$ } [illegible]

[illegible] $a=0$: $x^2 + y^2 - r^2 = 0$, $x = 0$ } [illegible] Chord.

$x^2 + y^2 - r^2 - \lambda x = 0$ } [illegible]

[illegible] Chord. imag. [illegible] $\pm r^2$

$m \pm$ $\qquad x^2 + y^2 + m = 0$

[illegible] $m -$ [illegible] Chord. [illegible]

$m +$ $\qquad$ conj. imag.

$x^2+y^2+m-\lambda x=0$ } [illegible]

$x^2+y^2-\lambda x+m=0$

[illegible] Absc. [illegible] $=\lambda$

[illegible]

$P(x'y')$ [illegible]

$xx'+yy'-\frac{\lambda}{2}(x+x')+m=0$ [illegible]

$xx'+yy'+m-\lambda\frac{x+x'}{2}=0$

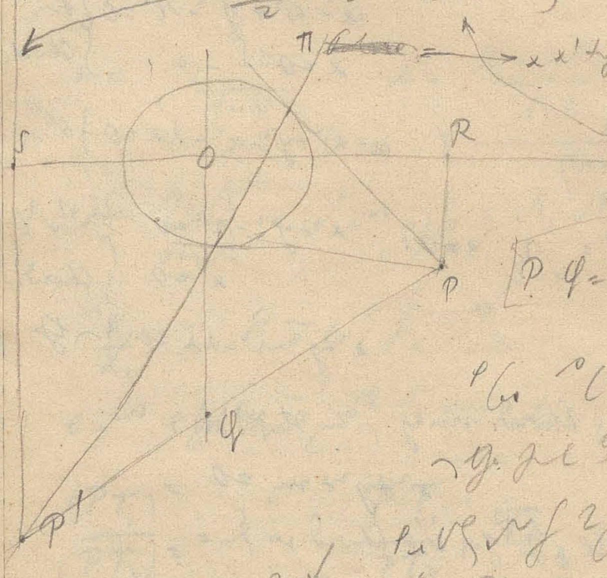
[illegible]:

$xx'+yy'+m=0$ } [illegible]

Π' ← $\frac{x+x'}{2}=0$ $x=-x'$

Π → $xx'+yy'+m=0$

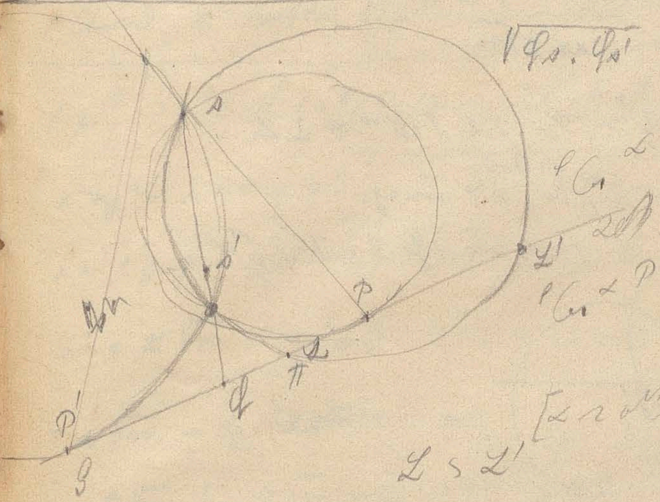
$OR=OS$



$[P\,Q=Q\,P'$

[illegible]

[illegible] P' [illegible] P



$\sqrt{\mathfrak{Q}s \cdot \mathfrak{Q}s'} = P\mathfrak{Q} = P'\mathfrak{Q}$

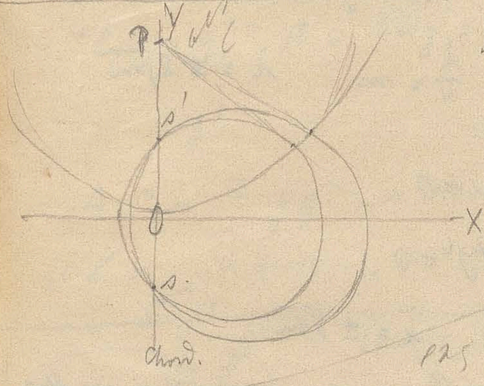
$\mathcal{L}$ s $\mathcal{L}'$

P, P' harm.

$(\mathcal{L}\mathcal{L}'PP') = -1$

Involution

$\overline{\mathfrak{Q}\mathcal{L}} \cdot \overline{\mathfrak{Q}\mathcal{L}'} = \overline{\mathfrak{Q}s}\;\overline{\mathfrak{Q}s'} = \overline{\mathfrak{Q}P}^2$



$x^2 + y^2 - \lambda x + m = 0$

Orthogon

= Potenz

$PT^2 = \mu^2 + m$

orthogonal:

$OP = \mu$

$(x^2 + \ldots) + (y - \mu)^2 = PT^2$

$x^2 + y^2 - 2\mu y + \mu^2 = \mu^2 + m$

$x^2 + y^2 - 2\mu y - m = 0 \qquad 2\mu = \lambda'$

$x^2 + y^2 - \lambda' y - m = 0$

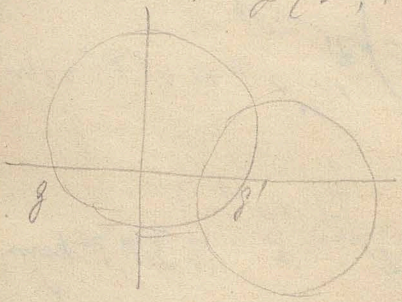
[illegible] Y ∞ = Centrale X ∞ = Chordale

[illegible] $y=0$ $x=\pm\sqrt{m}$

[illegible] 2 g ; [illegible] ⊥ X

[illegible] de Chord. [illegible], [illegible] imag.

g [illegible]



$x^2+y^2-\lambda x+m=0$ r = Rad.

$m=a^2+b^2-r^2$

$b=0$ $a=-\frac{\lambda}{2}$

$m=\frac{\lambda^2}{4}-r^2$

[illegible]: $\frac{\lambda^2}{4}=m$ $\lambda=\pm 2\sqrt{m}$

[illegible]:

$x^2+y^2\mp 2\sqrt{m}\,x+m=0$

$(x\mp\sqrt{m})^2+y^2=0$

$y=0$ $x=\pm\sqrt{m}$ [illegible]

[illegible] X ∞ de Abs. $\sqrt{m}$

[illegible]

= Grenzpuncte ; [illegible] s, s' [illegible]

a. orthog [illegible]

b. Chord. [illegible]

[illegible]

$x^2+y^2-\lambda x+m=0$

$xx'+yy'-\frac{\lambda}{2}(x+x')+m=0$ [illegible]

$x'=\pm\sqrt{m} \qquad y'=0$

$\pm x\sqrt{m}-\frac{\lambda}{2}(x\pm\sqrt{m})+m=0$

$\pm\sqrt{m}(x\pm\sqrt{m})-\frac{\lambda}{2}(x\pm\sqrt{m})=0$

$(x\pm\sqrt{m})\left(\pm\sqrt{m}-\frac{\lambda}{2}\right)=0.$

$x\pm\sqrt{m}=0$

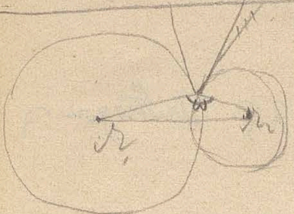
$x=\mp\sqrt{m}$ } [illegible]

[illegible] g [illegible] g' [illegible]

g' $\qquad$ g

[illegible] Δ [illegible] gg' [illegible] ∞ [illegible] conj. Δ [illegible]

[illegible]



[illegible] $\perp$ [illegible]

~~$\cos\omega=r_1^2$~~

$d^2=r_1^2+r_2^2-2r_1r_2\cos\omega$

$\cos\omega=$ ~~[illegible]~~ $\frac{r_1^2+r_2^2-d^2}{2r_1r_2}$

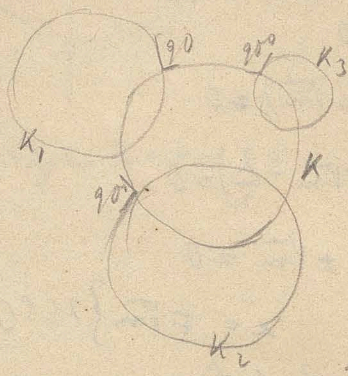
$=\frac{r_1^2+r_2^2-(a_1-a_2)^2-(b_1-b_2)^2}{2r_1r_2}$

$$= \frac{r_1^2 - a_1^2 - b_1^2 + r_2^2 - a_2^2 - b_2^2 + 2a_1a_2 + 2b_1b_2}{2r_1r_2}$$

$$= -\frac{c_1 + c_2 - 2(a_1a_2 + b_1b_2)}{2r_1r_2}$$

$$c_1 + c_2 = 2(a_1a_2 + b_1b_2)$$ } [illegible] $\perp$ [illegible]

[illegible]



[illegible] Orthog. [illegible]

[illegible]!:

$c + c_1 = 2(aa_1 + bb_1)$

$c + c_2 = 2(aa_2 + bb_2)$

$c + c_3 = 2(aa_3 + bb_3)$

$x^2 + y^2 - 2ax - 2by + c = 0$

[illegible] Orthog.

$1 \quad -2a \quad -2b \quad c$

[illegible]

$x^2 + y^2 - 2ax - 2by + c = 0$

$c_1 - 2aa_1 - 2bb_1 + c = 0$

$c_2 - 2aa_2 - 2bb_2 + c = 0$

$c_3 - 2aa_3 - 2bb_3 + c = 0$

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ c_1 & a_1 & b_1 & 1 \\ c_2 & a_2 & b_2 & 1 \\ c_3 & a_3 & b_3 & 1 \end{vmatrix} = 0$$ [illegible] Orthogon.

[illegible] Coord. [illegible]:

$$a = \frac{b_1(c_3-c_2)+b_2(c_1-c_3)+b_3(c_2-c_1)}{b_1(a_3-a_2)+b_2(a_1-a_3)+b_3(a_2-a_1)}$$

$$b = \frac{a_1(c_3-c_2)+a_2(c_1-c_3)+a_3(c_2-c_1)}{a_1(b_3-b_2)+a_2(b_1-b_3)+a_3(b_2-b_1)}$$

!!!

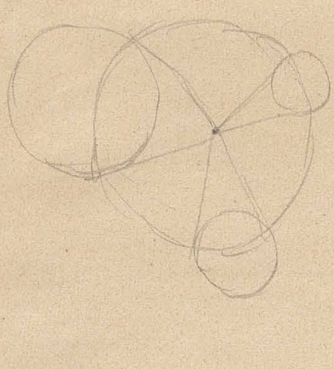
[illegible] = [illegible] Chord. [illegible] 3 [illegible]

$K_1 = 0 \quad K_2 = 0 \quad K_3 = 0$

$K_1 - K_2 = 0$
$K_2 - K_3 = 0$
$K_3 - K_1 = 0$

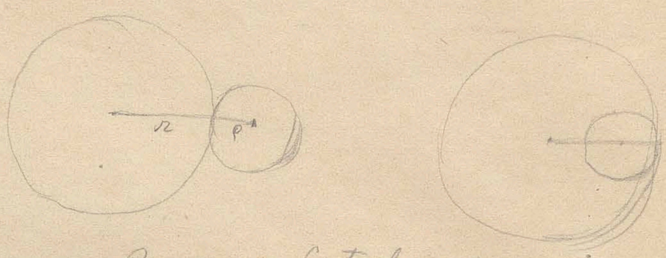
[illegible] Chord.
[illegible] $\Sigma \equiv 0$

[illegible] R = Radicalcentrum [illegible] 3 [illegible]



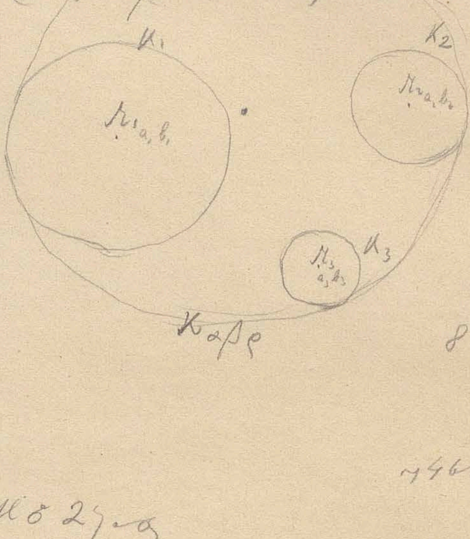
11/5

25/4 Apollonisches Problem



2 [illegible] Centrale $\rho \pm r$

$$(\alpha - a)^2 + (\beta - b)^2 = (\rho \pm r)^2$$



$$(\alpha - a_1)^2 + (\beta - b_1)^2 = (\rho \pm r_1)^2$$
$$(\alpha - a_2)^2 + (\beta - b_2)^2 = (\rho \pm r_2)^2$$
$$(\alpha - a_3)^2 + (\beta - b_3)^2 = (\rho \pm r_3)^2$$

8 [illegible]

[illegible]

+	+	+
−	−	−
−	+	+
+	−	−
+	−	+
−	+	−
+	+	−
−	−	+

[illegible] 2 [illegible]

[illegible] 8 [illegible]

$$\alpha^2 + \beta^2 - \rho^2 = t$$

$$a_i^2 + b_i^2 - r_i^2 = c_i \qquad i = 1, 2, 3$$

$$t + c_1 = 2a_1\alpha + 2b_1\beta + 2r_1\rho$$

$$t + c_2 = 2a_2\alpha + 2b_2\beta + 2r_2\rho$$
$$t + c_3 = 2a_3\alpha + 2b_3\beta + 2r_3\rho$$

$$\left.\begin{matrix}\alpha = \\ \beta = \\ \rho = \end{matrix}\right\} \text{lin. fc}(t)$$

$\alpha^2 + \beta^2 - \rho^2 = t^2$ [illegible] quadr. [illegible] t

$$\left.\begin{matrix} t_1 = \\ t_2 = \end{matrix}\right\}$$

[illegible]

$r_1 = 0$

$(\alpha - a_1)^2 + (\beta - b_1)^2 = \rho^2$

$(\alpha - a_2)^2 + (\beta - b_2)^2 = (\rho \pm r_2)^2$ ++

$(\alpha - a_3)^2 + (\beta - b_3)^2 = (\rho \pm r_3)^2$ +− } 4 Lös.

[illegible] r_1 [illegible] 4 [illegible] 2 [illegible]

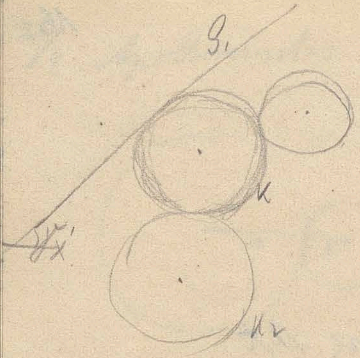
$r_2 = 0$

$(\alpha - a_1)^2 + (\beta - b_1)^2 = \rho^2$

$(\alpha - a_2)^2 + (\beta - b_2)^2 = \rho^2$ } 2 Lös.

$(\alpha - a_3)^2 + (\beta - b_3)^2 = (\rho \pm r_3)^2$

[illegible] $r = \infty$ [illegible]



$$x \cos\gamma_1 + y \sin\gamma_1 - p_1 = 0$$

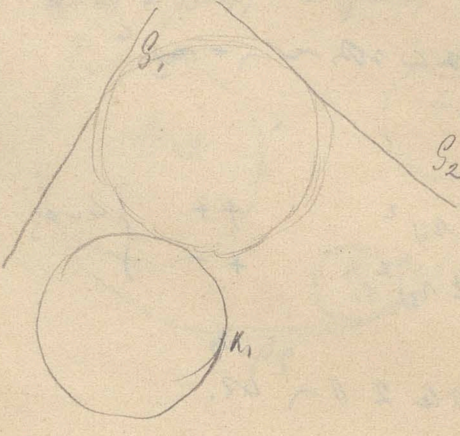
$$\alpha \cos\gamma_1 + \beta \sin\gamma_1 - p_1 = \text{[illegible]}$$
$$= \pm\rho$$

[illegible]:

$$\alpha \cos\gamma_1 + \beta \sin\gamma_1 - p_1 = \pm\rho$$

$$(\alpha - a_2)^2 + (\beta - b_2)^2 = (\rho \pm r_2)^2$$

$$(\alpha - a_3)^2 + (\beta - b_3)^2 = (\rho \pm r_3)^2$$



$$\alpha \cos\gamma_1 + \beta \sin\gamma_1 - p_1 = \pm\rho$$

$$\alpha \cos\gamma_2 + \beta \sin\gamma_2 - p_2 = \pm\rho$$

$$(\alpha - a_3)^2 + (\beta - b_3)^2 = (\rho \pm r_3)^2$$



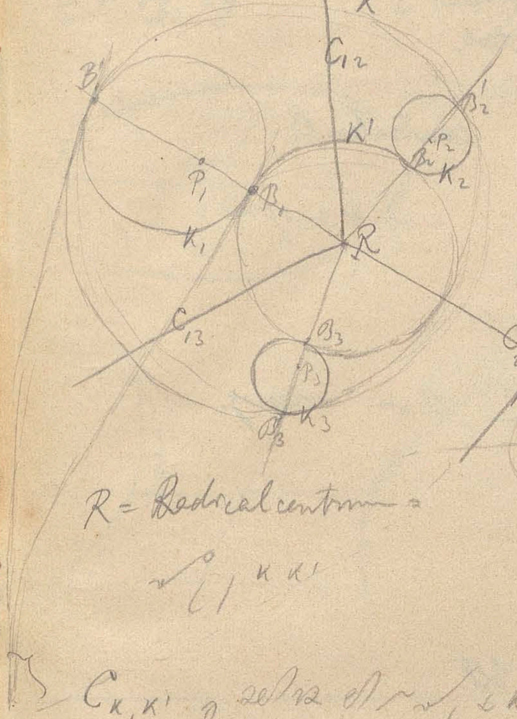
$$\alpha \cos\gamma_1 + \beta \sin\gamma_1 - p_1 = \pm\rho$$

$$\alpha \cos\gamma_2 + \beta \sin\gamma_2 - p_2 = \pm\rho$$

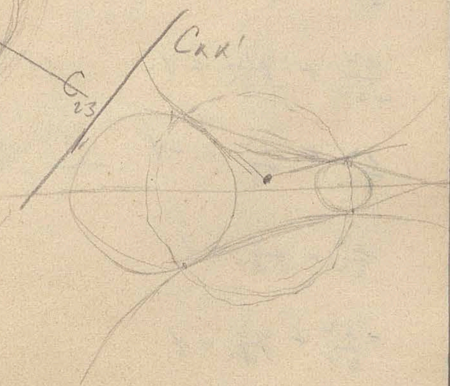
$$\alpha \cos\gamma_3 + \beta \sin\gamma_3 - p_3 = \pm\rho$$

4 [illegible]:

geom. Lösung:



+ + + } K K'
+ − −
+ − +
+ + −



R = Radicalcentrum [illegible]

[illegible] KK'

} $C_{K,K'}$ [illegible] $K_1 K_3$, $K_2 K_3$, $K_1 K_2$

[illegible]

[illegible] $C_{KK'}$ [illegible]

[illegible]

[illegible] R. Centr. R

[illegible] $P_1 P_2 P_3$ [illegible]

[illegible] $C'_{KK'}$ $C''_{KK'}$ $C'''_{KK'}$

25/5 C. f. ... centrale
acentrale

$\frac{x^2}{\alpha} + \frac{y^2}{\beta} = 1$ centralen C.

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...

$\frac{x^2}{-a^2} + \frac{y^2}{-b^2} = 1$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = -1$ ... imag.

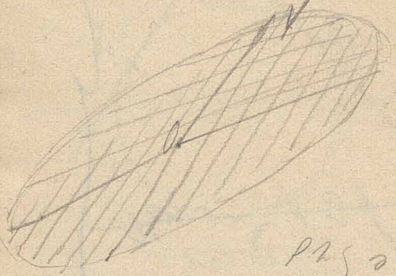
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ Ell.

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ Hyp.

... 2 conj. ...



X $\frac{x^2}{\alpha} + \frac{y^2}{\beta} = 1$

$a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + a_{33} = 0$

... quadr. ... x 2 ... y ...

$a_{11}x^2 + a_{22}y^2 + a_{33} = 0$ $-\frac{a_{33}}{a_{11}} = \alpha$ $-\frac{a_{33}}{a_{22}} = \beta$

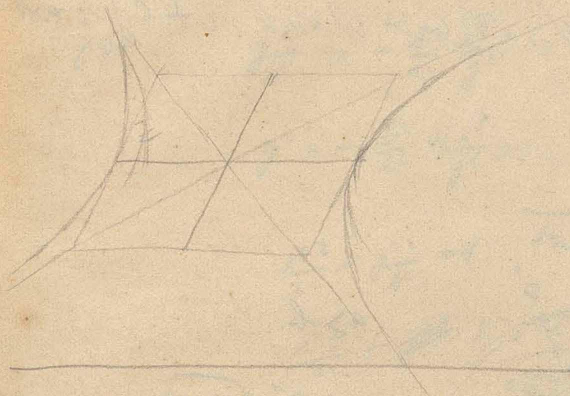
$\frac{x^2}{\alpha} + \frac{y^2}{\beta} = 1$

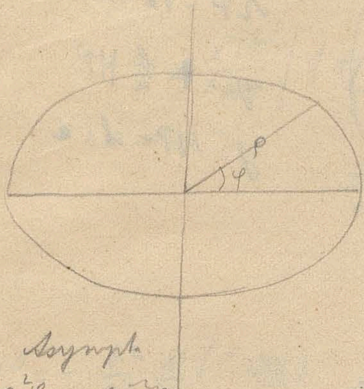
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ [illegible] $b^2 \pm$



Hyp. [illegible]

[illegible]





$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$x=\rho\cos\varphi$$

$$y=\rho\sin\varphi$$

$$\rho^2\left(\frac{\cos^2\varphi}{a^2}+\frac{\sin^2\varphi}{b^2}\right)=1$$

$$\frac{1}{\rho^2}=\frac{\cos^2\varphi}{a^2}+\frac{\sin^2\varphi}{b^2}$$

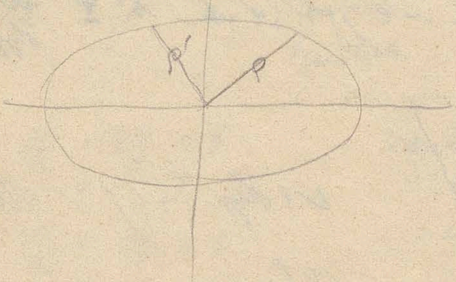
Asympt.

$$\frac{\cos^2\varphi}{a^2}+\frac{\sin^2\varphi}{b^2}=0$$

$$\operatorname{tg}^2\varphi=-\frac{b^2}{a^2}$$

$\operatorname{tg}\varphi=\pm\sqrt{-\frac{b^2}{a^2}}$ imag. [illegible] Asympt.

Hyp. $\left\{ \begin{matrix} b^2 \quad -b^2 \\ \pm\frac{b}{a} \end{matrix} \right\}$ [illegible]



$\varphi \quad \varphi + 90$

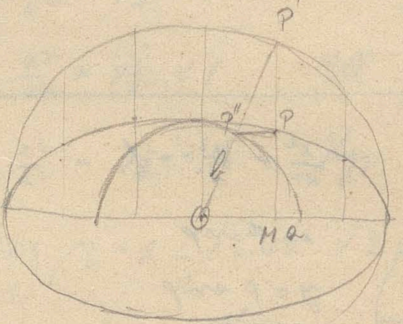
$$\frac{1}{\rho'^2} = \frac{\sin^2\varphi}{a^2} + \frac{\cos^2\varphi}{b^2}$$

$$\frac{1}{\rho^2} + \frac{1}{\rho'^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

[illegible]:
$a_{11} + a_{22}$ [illegible]

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$y = \pm \frac{b}{a}\sqrt{a^2 - x^2}$$



$a > b$

$MP' = \sqrt{a^2 - x^2}$

$y = \pm \frac{b}{a} MP'$

$y : MP' = b : a$

~~P'P : ...~~

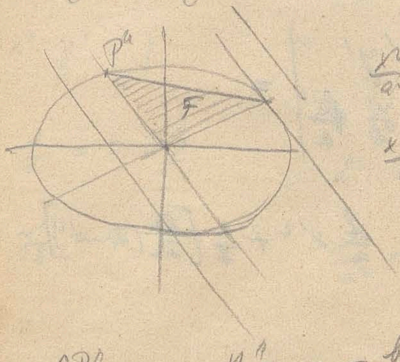
$OP'' : OP' = y : MP'$

$OP'' : OP' = b : a$

$OP'' : a = b : a$

$OP'' = b$

Rel. ∫ C. ‖ ^{n} ℓ cng. ℓℓ



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{xx'}{a^2} + \frac{yy'}{b^2} = 1 \quad \text{Polare } \ell \; x'y'$$

c $x'y'$ ~ ℓ $^{\ell c}$
b ^{2} ∫ ℓ pts.

OP''

$$\frac{y''}{x''} = -\frac{b^2}{a^2}\frac{x'}{y'}$$

$$y'' = -\frac{b^2}{a^2}\frac{x'x''}{y'}$$

$$\frac{x''^2}{a^2} + \frac{y''^2}{b^2} = 1$$

$$\frac{x''^2}{a^2} + b^4 \frac{x'^2 x''^2}{a^4 y'^2 b^2} = 1$$

$$\frac{x''^2}{a^2}\left[1 + \frac{b^2 x'^2}{a^2 y'^2}\right] = 1$$

$$\frac{x''^2}{a^2} \cdot \frac{a^2 b^2}{a^2 y'^2} = 1$$

$$\frac{x''^2}{a^2} = \frac{y'^2}{b^2}$$

$$\underbrace{\frac{x''}{a} = \pm \frac{y'}{b}}$$

$$y'' = -\frac{b^2}{a^2}\frac{x'}{y'}\left(\pm a \frac{y'}{b}\right)$$

$$y'' = \mp \frac{b}{a} x'$$

$$\underbrace{\frac{y''}{b} = \mp \frac{x'}{a}}$$

v e Hyp. b $^{\ell}$ b — $b\sqrt{-1}$

$P_1P_2 \triangle F$ = constant:

$$2F = \begin{vmatrix} x' & y' \\ x'' & y'' \end{vmatrix} = \begin{vmatrix} x' & y' \\ -a\frac{y'}{b} & \frac{bx'}{a} \end{vmatrix} =$$

$$= \frac{b}{a}x'^2 + \frac{a}{b}y'^2 = \frac{b}{a}x'^2 + \frac{a}{b}\left[\frac{b^2}{a^2}(a^2 - x'^2)\right]$$

$$= \frac{b}{a}a^2 = ab$$

$P \sum e^2$ conj. semid. = const.

$$d'^2 = x'^2 + y'^2$$

$$d''^2 = x''^2 + y''^2 = \frac{a^2y'^2}{b^2} + \frac{b^2x'^2}{a^2}$$

$$d'^2 + d''^2 = x'^2 + y'^2 + \frac{a^2y'^2}{b^2} + \frac{b^2x'^2}{a^2}$$

$$= x'^2 + \frac{b^2}{a^2}(a^2 - x'^2) + \frac{a^2}{b^2}\left(\frac{b^2}{a^2}(a^2 - x'^2)\right) + \frac{b^2x'^2}{a^2}$$

$$= a^2 + b^2$$

25/5 x' y' } ... d' d'' = ...

x'' y'' } ——— ...

$\frac{x'}{a} = \pm \frac{y''}{b}$

$\frac{y'}{b} = \mp \frac{x''}{a}$

$d' = x'^2 + y'^2$

$\frac{x'^2}{a^2} + \frac{y'^2}{b^2} = 1$

$y'^2 = \frac{b^2(1 - x'^2)}{a^2}$

$d'^2 = x'^2 + b^2 - \frac{b^2}{a^2} x'^2$

$= x'^2 \left(1 - \frac{b^2}{a^2}\right) + b^2$

$= x'^2 \frac{a^2 - b^2}{a^2} + b^2$

$a^2 - b^2 = c^2 =$ Exc.

$\frac{c}{a} = e =$ num. Exc.

$d'^2 = b^2 + e^2 x'^2$

$d''^2 = \frac{b^2}{a^2} x'^2 + a^2\left(1 - \frac{x'^2}{a^2}\right)$

$= a^2 - x'^2$

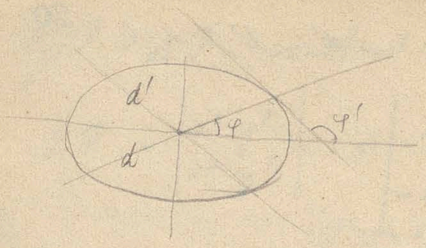
$= a^2 - x'^2\left(1 - \frac{b^2}{a^2}\right)$

$d''^2 = a^2 - e^2 x'^2$

$d'^2 + d''^2 = a^2 + b^2$ Ellr

Σ e 2 conj. Halb = const.

$d'^2 - d''^2 = a^2 - b^2$ } Hyperbel



$\operatorname{tg}\varphi = +\frac{y'}{x'}$

$\frac{xx'}{a^2} + \frac{yy'}{b^2} = 1$

$\operatorname{tg}\varphi' = -\frac{b^2 x'}{a^2 y'}$

$\operatorname{tg}\varphi \operatorname{tg}\varphi' = -\frac{b^2}{a^2}$ [illegible]

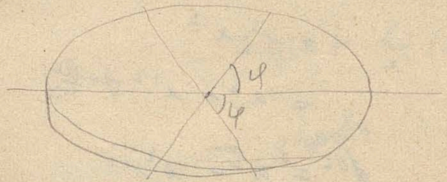
$\operatorname{tg}\varphi \operatorname{tg}\varphi' = \frac{b^2}{a^2}$ Hyperbel [illegible]

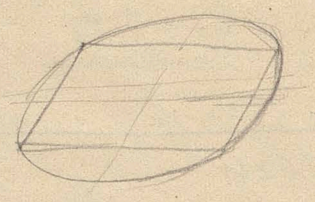
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{1}{r^2} = \frac{\cos^2\varphi}{a^2} + \frac{\sin^2\varphi}{b^2}$

[illegible]

[illegible]





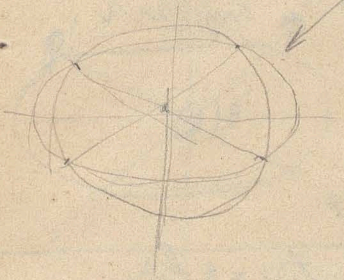
[illegible]

$d' = d''$

$2d'^2 = a^2 + b^2$

$d'^2 = \frac{a^2+b^2}{2}$

$d' = \sqrt{\frac{a^2+b^2}{2}}$

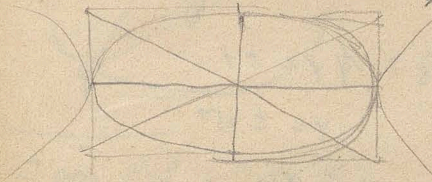


$\operatorname{tg}\varphi \operatorname{tg}\varphi' = -\frac{b^2}{a^2}$

$\varphi' = \varphi$

$\operatorname{tg}^2\varphi = \frac{b^2}{a^2}$

$\operatorname{tg}\varphi = \pm\frac{b}{a}$



= Asympt. [illegible] Hyp. [illegible]

[illegible] Hyp. [illegible]

[illegible]

[illegible] $d'^2 = \sqrt{\frac{a^2-b^2}{2}}$

[illegible] Asympt.

$d' = \infty \quad d'' = \infty$

[illegible]

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ [illegible]

$\frac{x^2}{d'^2} + \frac{y^2}{d''^2} = 1$. $d' = d''$

$x^2 + y^2 = d'^2$ [illegible]

[illegible]

$d'^2 - d''^2 = a^2 - b^2$

$d' = d''$

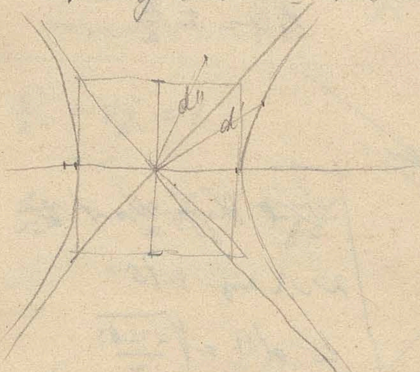
$d'^2 - d''^2 = 0 \qquad a = b$

$d' = d''$

ω – Hyp. [illegible]

$\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$

$x^2 - y^2 = a^2 \quad = \ldots$ Hyp.



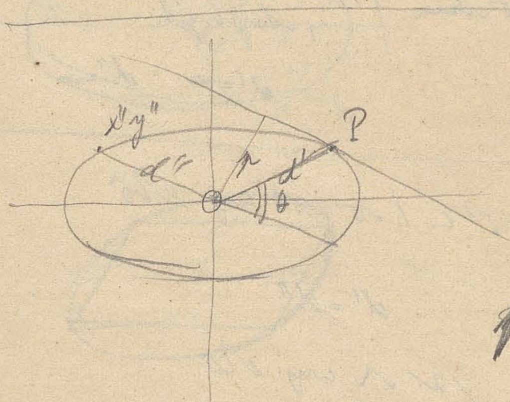
$tg\,\varphi = \pm \frac{b}{a}$

$b = a$

$tg\,\varphi = \pm 1$

$\varphi = \pm 45^\circ$

as. $\perp$



$\frac{xx'}{a^2} + \frac{yy'}{b^2} = 1$

$$\frac{\frac{xx'}{a^2} + \frac{yy'}{b^2} - 1}{\sqrt{\frac{x'^2}{a^4} + \frac{y'^2}{b^4}}}$$

$$r = \frac{1}{\sqrt{\frac{x'^2}{a^4} + \frac{y'^2}{b^4}}}$$

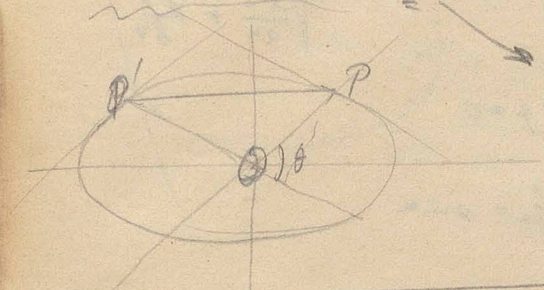
$$= \frac{ab}{\sqrt{\frac{b^2x'^2}{a^2} + \frac{a^2y'^2}{b^2}}} = \frac{ab}{\sqrt{y''^2 + x''^2}} = \frac{ab}{d''}$$

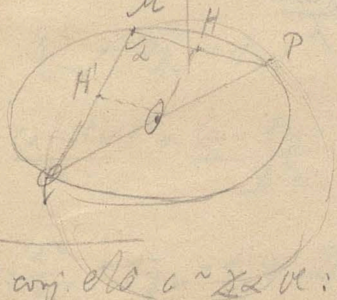
$$p = \frac{ab}{d''}$$

$$\sin\theta = \frac{p}{d'} = \frac{ab}{d'd''}$$

$$d'd''\sin\theta = ab$$

$= \Delta\, POP' = \text{const}$



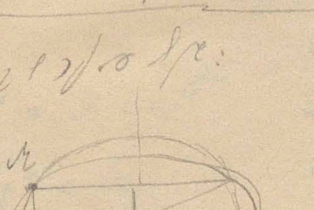


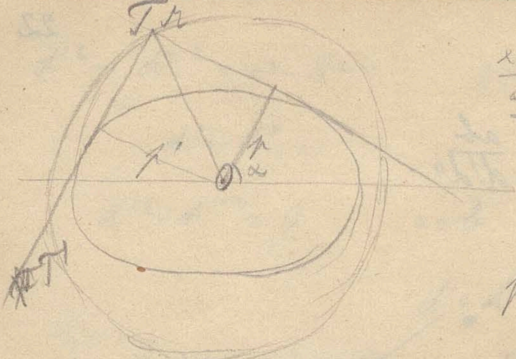
MP, MQ = Supplementar-Sehnen [illegible] 2 conj. D.

2 conj. [illegible]:

[illegible] QP [illegible] PQ [illegible] $\measuredangle\alpha$

[illegible]:





$$\frac{\frac{xx'}{a^2}+\frac{yy'}{b^2}-1}{\sqrt{\frac{x'^2}{a^4}+\frac{y'^2}{b^4}}}=0$$

$$p=\frac{1}{\sqrt{\frac{x'^2}{a^4}+\frac{y'^2}{b^4}}}$$

$$x\cos\alpha + y\sin\alpha - p = 0$$

$$p\frac{x'}{a^2}=\cos\alpha \qquad p\frac{y'}{b^2}=\sin\alpha$$

$$\frac{x'}{a}=\frac{a\cos\alpha}{p} \qquad \frac{y'}{b}=\frac{b\sin\alpha}{p}$$

$$1=\frac{a^2\cos^2\alpha}{p^2}+\frac{b^2\sin^2\alpha}{p^2}$$

$$p^2=a^2\cos^2\alpha+b^2\sin^2\alpha$$

$$p'^2=a^2\cos^2(90+\alpha)+b^2\sin^2(90+\alpha)$$

$$p'^2=a^2\sin^2\alpha+b^2\cos^2\alpha$$

$$p^2+p'^2=a^2+b^2=OM^2$$

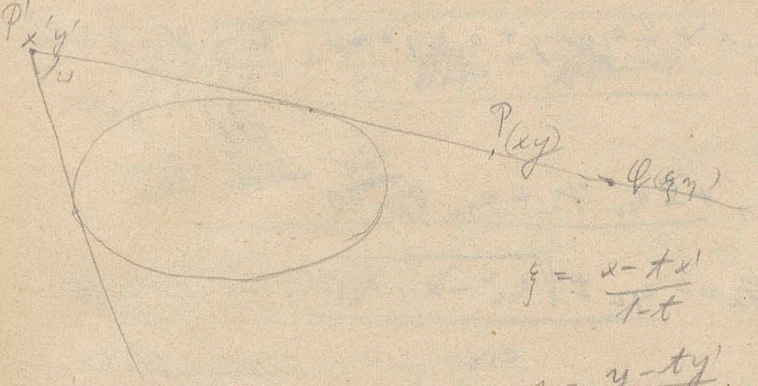
$OM=\sqrt{a^2+b^2}$ [illegible]

Hyp: $OM=\sqrt{a^2-b^2}$ [illegible] $a>b$

$a=b$ [illegible] Asympt. [illegible] ⊥ [illegible]

[illegible]

[illegible]?



$$\xi = \frac{x - t x'}{1-t}$$

$$\eta = \frac{y - t y'}{1-t}$$

$$\frac{\xi^2}{a^2} + \frac{\eta^2}{b^2} = 1$$

$$\frac{(x - t x')^2}{a^2} + \frac{(y - t y')^2}{b^2} = (1-t)^2$$

$$\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right) - 2t\left(\frac{x x'}{a^2} + \frac{y y'}{b^2} - 1\right) +$$

$$+ t^2\left(\frac{x'^2}{a^2} + \frac{y'^2}{b^2} - 1\right) = 0.$$

$t = 2$ [illegible]

[illegible] < [illegible]

[illegible]:

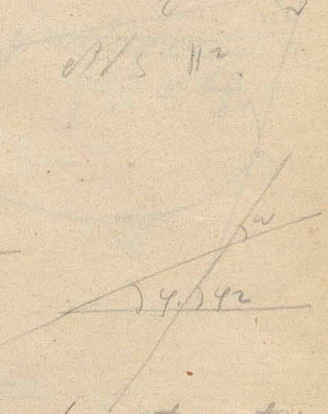
$$\left(\frac{x x'}{a^2} + \frac{y y'}{b^2} - 1\right)^2 - \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)\left(\frac{x'^2}{a^2} + \frac{y'^2}{b^2} - 1\right) = 0$$

[illegible] P'P [illegible] etc. — } [illegible]

$a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_{13}x + 2a_{23}y + a_{33} = 0$

[illegible]

$a_{11}x^2 + 2a_{12}xy + a_{22}y^2 = 0$ [illegible]



$a_{11} + 2a_{12}\frac{y}{x} + a_{22}\left(\frac{y}{x}\right)^2 = 0$

$\frac{y}{x} = \frac{-a_{12} \pm \sqrt{a_{12}^2 - a_{11}a_{22}}}{a_{22}}$

$\tan\omega = \frac{\tan\varphi_1 - \tan\varphi_2}{1 + \tan\varphi_1 \tan\varphi_2}$

$\tan\omega = \frac{\frac{2\sqrt{a_{12}^2 - a_{11}a_{22}}}{a_{22}}}{1 + \frac{a_{11}a_{22}}{a_{22}^2}} = \frac{2\sqrt{a_{12}^2 - a_{11}a_{22}}}{a_{22} + a_{11}}$

~~$\frac{\partial x'}{\partial y}$ $a_{11} = \frac{x'^2}{a^4} - \frac{1}{a^2}\left(\frac{x'^2}{a^2} + \frac{y'^2}{b^2} - 1\right)$~~

$a_{11} = -\frac{1}{a^2}\left(\frac{y'^2}{b^2} - 1\right)$

$a_{22} = -\frac{1}{b^2}\left(\frac{x'^2}{a^2} - 1\right)$

$a_{12} = \frac{x'y'}{a^2b^2}$

$$\operatorname{tg}\omega = \pm\frac{2\sqrt{\frac{x'^2y'^2}{a^4b^4} - \frac{1}{a^2b^2}\left(\frac{y'^2}{b^2}-1\right)\left(\frac{x'^2}{a^2}-1\right)}}{\frac{1}{a^2}\left(\frac{y'^2}{b^2}-1\right)\frac{1}{a^2}+\frac{1}{b^2} = \frac{x'^2}{a^2b} - \frac{y'^2}{ab^2}}$$

$$= \frac{2\sqrt{\left(\frac{y'^2}{b^2}+\frac{x'^2}{a^2}\right) - 1}}{\frac{1}{ab}\ \frac{x'^2}{\ } - \frac{y'^2}{\ }}$$

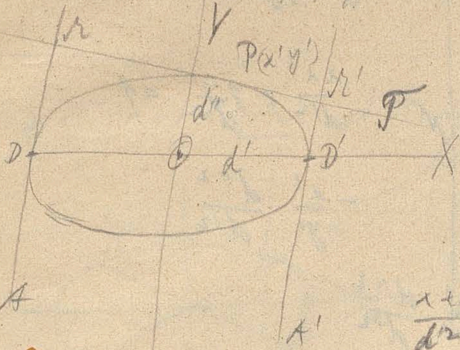
$$= \frac{2ab\sqrt{\frac{x'^2}{a^2}+\frac{y'^2}{b^2}-1}}{a^2+b^2-(x'^2+y'^2)}$$

= ⊥ [illegible]

[illegible] ⊥ [illegible]

$x'^2+y'^2 = a^2+b^2$ = ◠

[illegible]



e [illegible] DM × D'M' = [illegible]

$$\frac{x^2}{d'^2}+\frac{y^2}{d''^2}=1$$

$$\frac{xx'}{d'^2}+\frac{yy'}{d''^2}=1$$

$d'r'$: $x = d'$, $y = \dots$

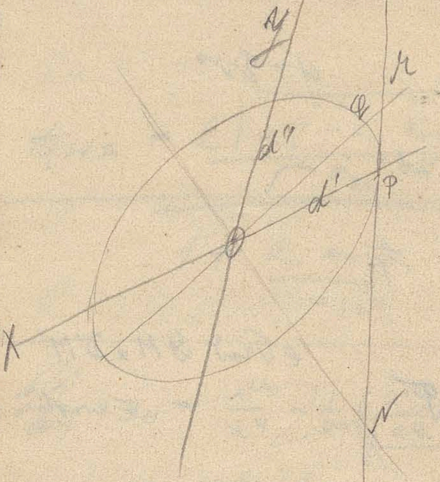
$$\frac{d'x}{d'^2} + \frac{B'r'y'}{d''^2} = 1$$

$$B'r' = \frac{d''^2}{y'}\left[1 - \frac{x'}{d'}\right]$$

$$Br = \frac{d''^2}{y'}\left[1 + \frac{x'}{d'}\right]$$

$$Br \cdot B'r' = \frac{d''^4}{y'^2}\left[1 - \frac{x'^2}{d'^2}\right]$$

$$Br \cdot B'r' = d''^2$$



$\overline{NP}\ \overline{Pr} = d''^2$

$$\frac{x^2}{d'^2} + \frac{y^2}{d''^2} = 1$$

$y = \frac{y'}{x'} x$ — Or

$$\frac{xx'}{d'^2} + \frac{yy'}{d''^2} = 1$$

$$-\frac{x'}{y'}\frac{d''^2}{d'^2}$$

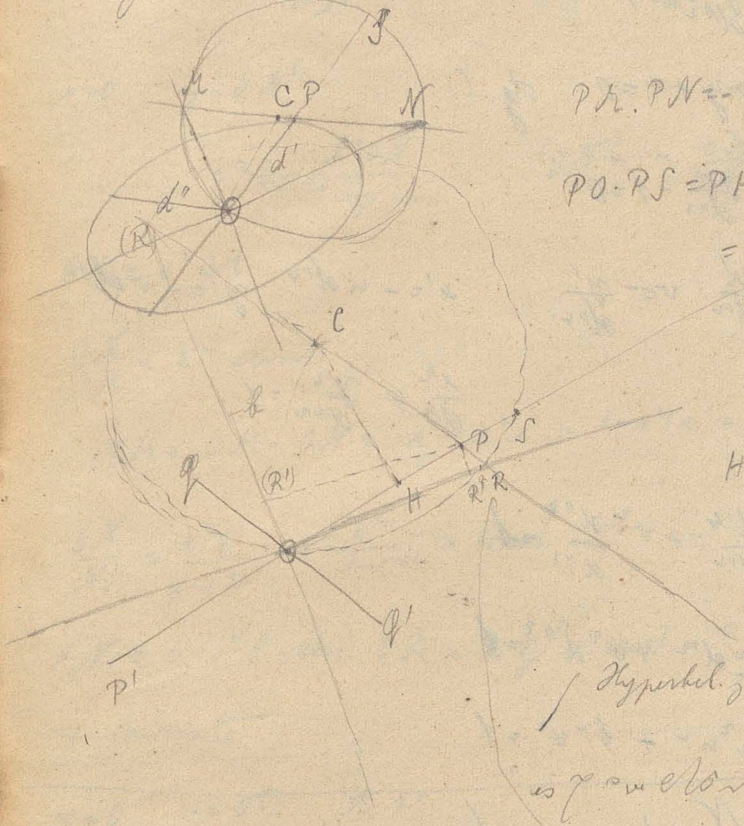
ON — $y = -\frac{x'}{y'}\frac{d''^2}{d'^2}$

Or : $x = d'$; $y = Pr = \frac{y'}{x'} \cdot d'$

ON — $y = PN = -\frac{x'}{y'}\frac{d''^2}{d'^2} d'$

$PR \cdot PN = -d''^2$

[illegible] 2 conj. [illegible]



$PR \cdot PN = -d''^2$

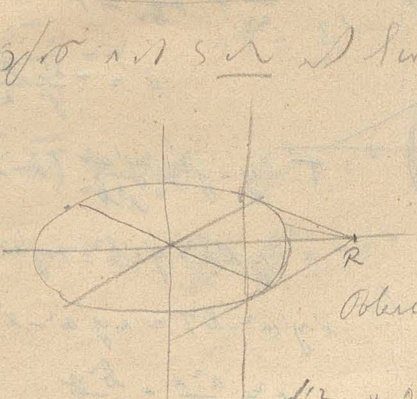
$PO \cdot PS = PM \cdot PN$
$= -d''^2$

$HC \perp PP'$

Hyperbel [illegible]

[illegible]

[illegible]



$\frac{y y'}{d'^2} + \frac{x x'}{d''^2} = 1$

$x' = OR \quad y' = 0$

Oben: $\frac{x\,OR}{d'^2} = 1 \qquad x = \frac{d'^2}{OR}$

$d'^2 = x\,OR$

$$\frac{x^2}{d'^2}+\frac{y^2}{d''^2}=1$$

$$ux+vy+1=0 \quad \text{tg. C.}$$

$$\frac{xx'}{d'^2}+\frac{yy'}{d''^2}-1=0$$

$$u=-\frac{x'}{d'^2} \quad v=-\frac{y'}{d''^2} \qquad x'=-ud'^2 \quad y'=-vd''^2$$

$$\frac{x'^2}{d'^2}+\frac{y'^2}{d''^2}=1$$

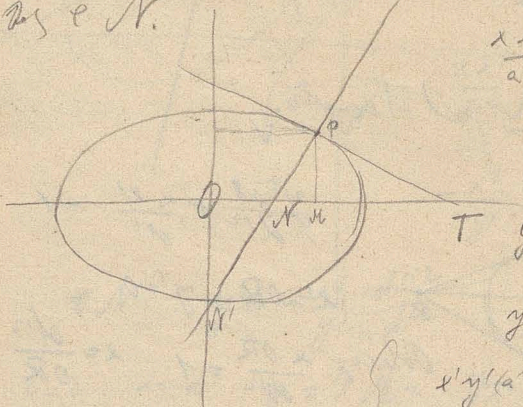
$$u^2\frac{d'^4}{d'^2}+v^2\frac{d''^4}{d''^2}=1$$

$$u^2d'^2+v^2d''^2=1$$

$$a^2u^2+b^2v^2=1$$

---

tg e N.



$$\frac{xx'}{a^2}+\frac{yy'}{b^2}=1$$

$$A=-\frac{x'}{y'}\frac{b^2}{a^2}$$

T $$y-y'=\frac{y'a^2}{x'b^2}(x-x')$$

$$yx'b^2-y'x'b^2=xy'a^2-x'y'a^2$$

$$x'y'(a^2-b^2)=xy'a^2-x'yb^2$$

$$c^2=\frac{a^2x}{x'}-\frac{b^2y}{y'}$$

$y=0 \quad x=ON$

$$c^2 = \frac{a^2 ON}{x'} \qquad ON = \frac{c^2 x'}{a^2} = e^2 x'$$

$x=0 \quad y=ON'$

$$c^2 = -\frac{b^2 ON'}{y'} \qquad ON' = -\frac{c^2 y'}{b^2}$$

$ON = e^2 OM$

$\frac{ON}{OM} = e^2 = \text{const.}$

$NM = OM - ON = x' - e^2 x' = (1-e^2)x' =$

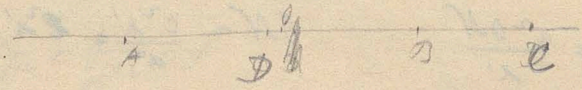
$\frac{ON}{NM} = \frac{e^2 x'}{(1-e^2)x'} = \frac{e^2}{1-e^2} = \text{const.}$

l. N. e C. d. l. Absc. OM = 2 g. a l. const. 1° o Ordinate

$y=0 \quad x=OT \quad$ tg. ns

$\frac{OT\, x'}{a^2} = 1 \qquad OT = \frac{a^2}{x'} \qquad \Big| \qquad OT' = \frac{b^2}{y'}$

$$\left\{ \begin{aligned} \overline{ON}.\overline{OT} &= \frac{c^2 x'}{a^2} \cdot \frac{a^2}{x'} = c^2 \\ ON'.OT' &= -\frac{c^2 y'}{b^2}\, \frac{b^2}{y'} = -c^2 \end{aligned} \right\}$$



(ABCD) = −1   OA = OB

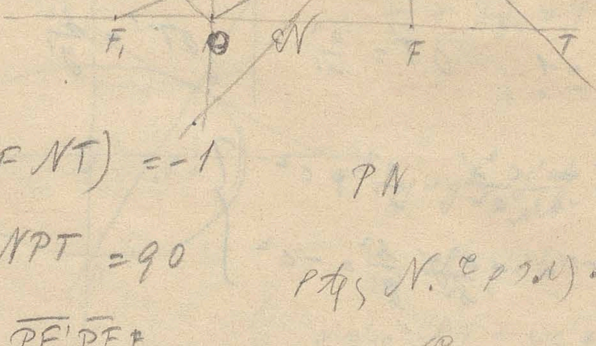
$\overline{OC} \cdot \overline{OD} = \left(\frac{AB}{2}\right)^2$

Involution

$F'O = OF'' = \sqrt{c^2} = c$ = G (e Invol.

$\varphi' O = O\varphi'' = \sqrt{c^2} = ci$

F, F'
φ, φ' } G



(F' F N T) = −1   PN

∡ NPT = 90

$\overline{PF'}\,\overline{PF}$

$OT = \frac{a^2}{x'}$ $c^2 = a^2 - b^2$ ~~Hyper~~

$c < a$

$x' < a$

$\left(\frac{x'}{a}\right)^2 + \left(\frac{y'}{b}\right)^2 = 1$

$\frac{a}{x'} > 1$ $\frac{a^2}{x'} > a$ $OT > a$ T [illegible]

$ON = \frac{c^2 x'}{a^2} = c \frac{c}{a} \frac{x'}{a} < c$

$<1 \quad <1$

$c^2 = a^2 + b^2$

$c > a$ $\frac{x'^2}{a^2} - \frac{y'^2}{b^2} = 1$

$\frac{x'^2}{a^2} > 1$ $x' > a$

$\frac{a}{x'} < 1$

$\frac{a^2}{x'} < a$

$OT < a$ $ON = \frac{c^2 x'}{a^2} = c \cdot \frac{c}{a} \cdot \frac{x'}{a} > c$ [illegible] FP'

$>1 \quad >1$

[illegible] Hyp [illegible]

$NM = x' - e^2 x'$

$PN^2 = y'^2 + x'^2(1-e^2)^2 = y'^2 + x'^2\left(1 - \frac{c^2}{a^2}\right)^2 =$

$= y'^2 + x'^2 \frac{b^4}{a^4} = \frac{b^2}{a^2}\left(\frac{a^2 y'^2}{b^2} + \frac{b^2 x'^2}{a^2}\right)$

d'^2

$OP =$ $PN = \frac{b d'}{a}$

$PN' = \frac{a}{b} d''$ [illegible]

$PN \cdot PN' = d''^2$

$PN : PN' = \frac{b^2}{a^2}$ const. [illegible]

$PF = r$

$PF' = r'$ $\qquad r^2 = (x'-c)^2 + y'^2$

$$= x'^2 - 2cx' + c^2 + \frac{b^2}{a^2}(a^2 - x'^2)$$

$$= x'^2\left(1 - \frac{b^2}{a^2}\right) - 2cx' + c^2 + b^2$$

$$= \frac{x'^2 c^2}{a^2} - 2cx' + a^2$$

$$= \left(\frac{x'c}{a} - a\right)^2 = (a - x'e)^2$$

$r = \pm(a - ex')$ [illegible]

$r' = (a + ex')$ [illegible]

$r + r' = 2a$ [illegible] const.

[illegible] Hyp. [illegible]

$r = ex' - a$

$r' = ex' + a$

$r' - r = 2a$ [illegible]

$OT \cdot ON = c^2$ $\quad$ FF' [illegible]

$OT' \cdot ON' = -c^2$ $\quad$ $\Phi\Phi'$ imag.

$ON = c^2 x'$ $\quad$ $ON' = -$

$OT = \frac{a^2}{x'}$ $\quad$ $OT' = \frac{b^2}{y'}$

[illegible]

[illegible]:

$$\left(\frac{xx'}{a^2} + \frac{yy'}{b^2} - 1\right)^2 - \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)\left(\frac{x'^2}{a^2} + \frac{y'^2}{b^2} - 1\right) = 0$$

[illegible]

$y' = 0 \quad x' = c$

$$\left(\frac{cx}{a^2} - 1\right)^2 - \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)\left(\frac{c^2}{a^2} - 1\right) = 0$$

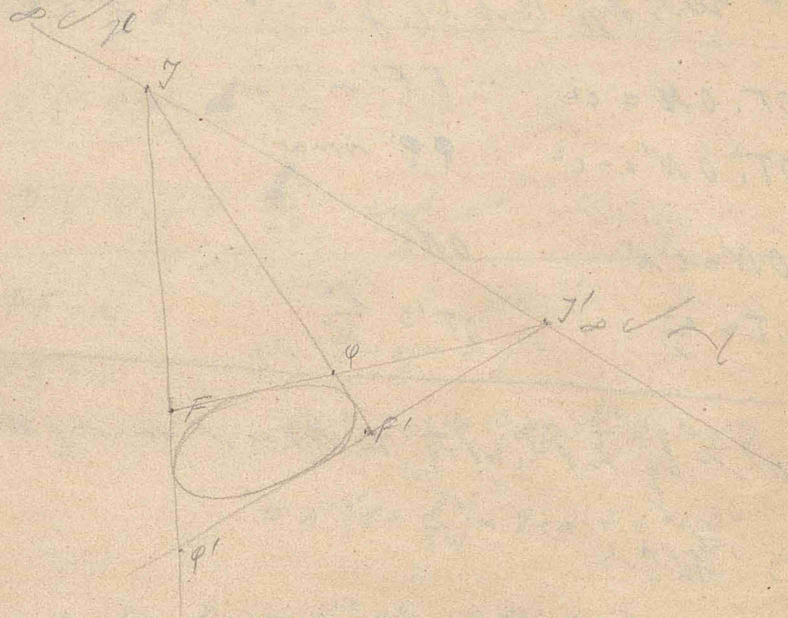
[illegible] $^2 x$ [illegible] $= 0$ [illegible]

$$\frac{c^2x^2}{a^4} - \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)\left(\frac{c^2}{a^2} - 1\right) = 0$$

$$\frac{c^2x^2}{a^2} + \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) b^2 = 0$$

$$x^2 \frac{(c^2+b^2)}{a^2} + y^2 = 0$$

$x^2 + y^2 = 0 \equiv$ [illegible]



[illegible]

[illegible]

$FF' =$ [illegible]

$\varphi\varphi' =$ [illegible]

[illegible]:

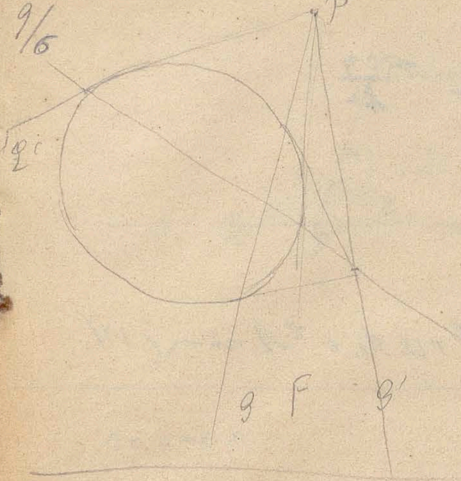


[illegible]

[illegible] imag.

46 [illegible] = [illegible]

9/6



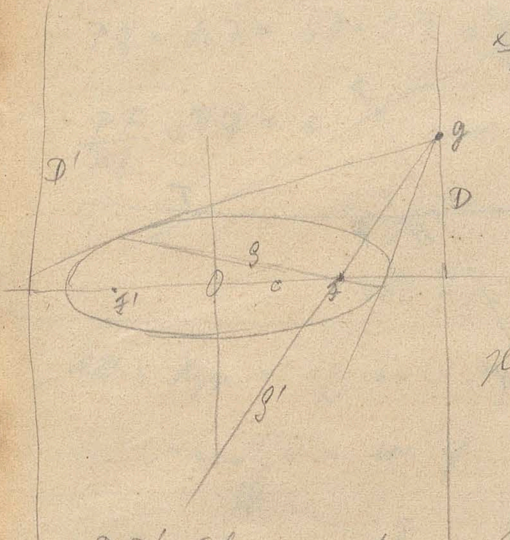
[illegible]

[illegible] — Involution

$(QF\, g g') = -1$

[illegible]

$g' \perp g.$



$$\frac{x x'}{a^2} + \frac{y y'}{b^2} = 1$$

$y' = 0 \qquad x' = \pm c$

$$\pm \frac{c x}{a^2} = 1$$

$$x = \pm \frac{a^2}{c} = \pm \frac{a}{e}$$

[illegible] $\perp$ X[illegible]

D, D' Polaren [illegible], [illegible] = Directrix

[illegible] Invol. conj. [illegible]

BH.
$g' \perp g$:

$g : \xi, \eta \qquad \xi = \frac{a^2}{c}$

$$g : \frac{x \xi}{a^2} + \frac{y \eta}{b^2} = 1$$

$$\frac{x}{c} + \frac{y \eta}{b^2} = 1$$

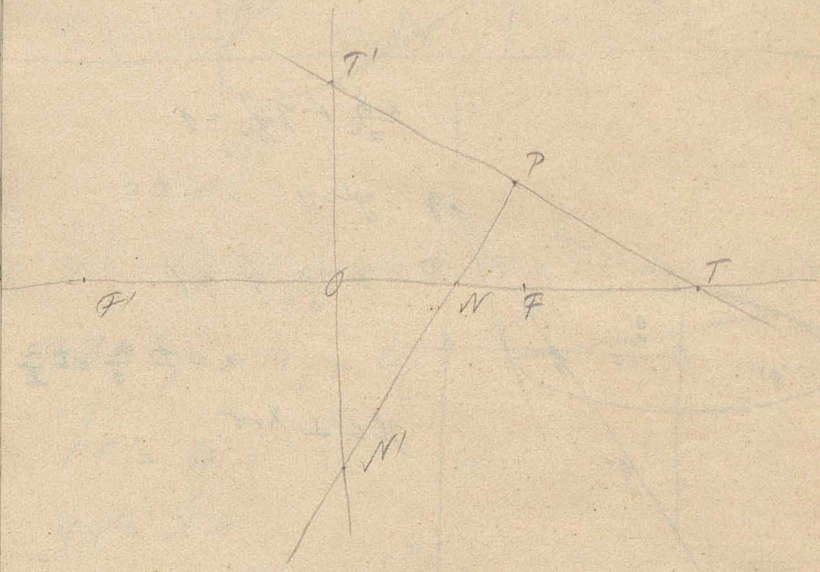
$$- \frac{b x}{c \eta} = \text{[illegible] Const.}$$

g': ... $\dfrac{\eta - 0}{\frac{a^2}{c} - c} = \dfrac{c\eta}{a^2 - c^2} = \dfrac{c\eta}{b^2}$

$g' \perp g$.

[illegible]

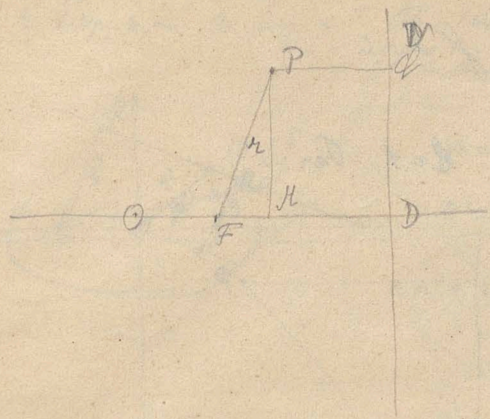
rechtw. [illegible]



$ON \cdot OT = c^2$ $\qquad OF' = c$

$ON' \, OT' = -c^2$

$\varphi O = O\varphi' = c\sqrt{-1}$ [illegible]



$OD = \frac{a^2}{c} = \frac{a}{e}$

[illegible] $\frac{a}{c} > 1$

$x < a$

$x < OD$

$r = a - e x$

$PQ = MD = OD - OM = \frac{a}{e} - x = \frac{a - ex}{e}$

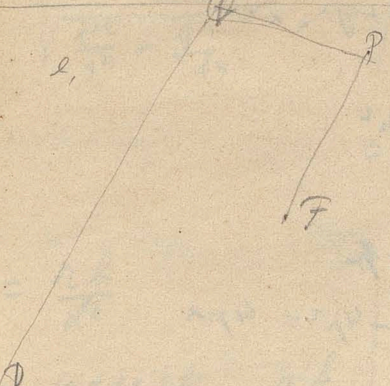
$\frac{PF}{PQ} = e$ [illegible]

[illegible] Ell. [illegible]

[illegible] Hyp.

Ell. [illegible] Hyp [illegible]

[illegible]



$D \quad Ax + By + C = 0$

$F \quad m, n$

$\frac{PF}{PQ} = e$

$\frac{PF^2}{PQ^2} = e^2$

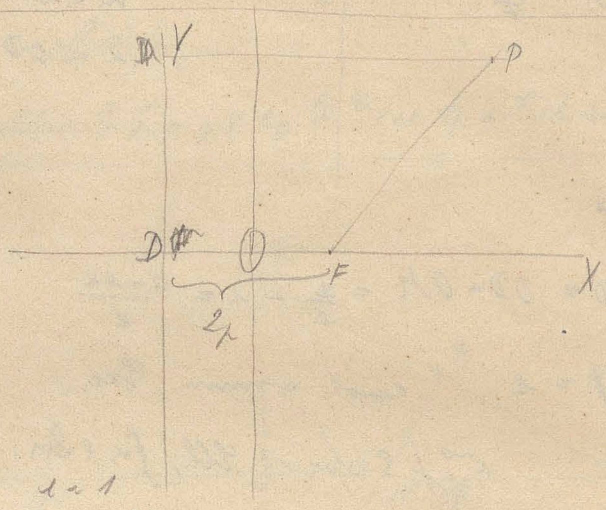
$PF^2 - e^2 PQ^2 = 0$

$(x-m)^2 + (y-n)^2 - \frac{(Ax+By+C)^2}{A^2+B^2} e^2 = 0$

~~[illegible]~~ = [illegible]

Ell. $e < 1$ $\qquad$ $e = 1$ Par.

Hyp. $e > 1$



$e = 1$

$PF = PG$

$PF^2 = PG^2$

$(x - 2p)^2 + y^2 = x^2$

$$x^2 - 4px + 4p^2 + y^2 = x^2$$

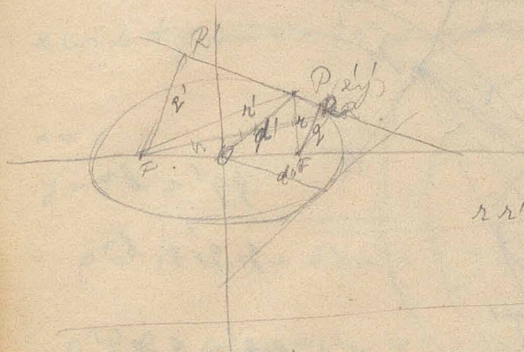
$$y^2 = 4px - 4p^2$$

$DO = OF = p \qquad y = y$

$$x = x + p$$

$$y^2 = 4p(x + p) - 4p^2 = 4px$$

[illegible]



$r = (a - ex'$

$r' = a + ex'$

$rr' = a^2 - e^2x'^2 = d''^2 = \frac{b^2x'^2}{a^2} + \frac{a^2y'^2}{b^2}$

$$rr' = a^2 - \frac{c^2}{a^2}x'^2 = \frac{a^4 - (a^2-b^2)x'^2}{a^2} = \frac{a^2(a^2-x'^2)+b^2x'^2}{a^2} =$$

$$= (a^2 - x'^2) + \frac{b^2}{a^2}x'^2 = \frac{a^2}{b^2}y'^2 + \frac{b^2}{a^2}x'^2 = d''^2$$

[illegible]

$$\frac{xx'}{a^2} + \frac{yy'}{b^2} - 1 = 0$$

$$\frac{\frac{xx'}{a^2} + \frac{yy'}{b^2} - 1}{\sqrt{\frac{x'^2}{a^4} + \frac{y'^2}{b^4}}} = 0$$

$$q: \frac{1 - \frac{cx'}{a^2}}{\sqrt{\frac{x'^2}{a^4} + \frac{y'^2}{b^4}}} =$$

$$= \frac{\left(1 - \frac{ex'}{a}\right)ab}{\sqrt{\frac{b^2x'^2}{a^2} + \frac{a^2y'^2}{b^2}}} = \frac{(a - ex')b}{d''}$$

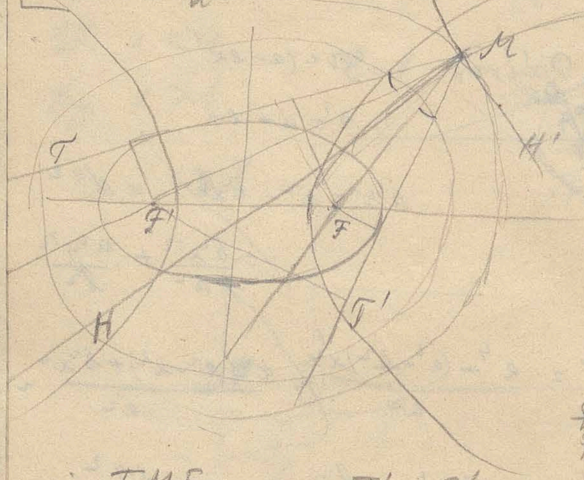
$$= \frac{rb}{d''}$$

$$q' = \frac{(a+ex')b}{d''} = \frac{r'b}{d''}$$

$$\frac{q}{r} = \frac{q'}{r'} = \frac{b}{d''} = \sin RPF = \sin R'PF'$$

$RPF = R'PF'$

$$qq' = \frac{b^2 r^2}{d^2} = b^2$$



$$qq' = b^2$$

$$q_1 q_1' = b^2$$

$$qq' = q_1 q_1'$$

$$\frac{q}{q_1} = \frac{q_1'}{q'}$$

$$\frac{\sin TMF}{\sin T'MF} = \frac{\sin T'MF'}{\sin TMF'}$$

[illegible]



$$x \cos\alpha + y \sin\alpha - p = 0$$

$$\frac{x \cos\alpha}{p} + y\frac{\sin\alpha}{p} - 1 = 0$$

$$\frac{x x'}{a^2} + \frac{y y'}{b^2} - 1 = 0$$

$$\frac{\cos\alpha}{p} = \frac{x'}{a^2} \quad \frac{\sin\alpha}{p} = \frac{y'}{b^2}$$

$$\frac{a \cos\alpha}{p} = \frac{x'}{a}$$

$$\frac{b \sin\alpha}{p} = \frac{y'}{b}$$

$$\frac{a^2 \cos^2\alpha}{p^2} + \frac{b^2 \sin^2\alpha}{p^2} = 1$$

$$p = \sqrt{a^2\cos^2\alpha + b^2\sin^2\alpha}$$

$$x\cos\alpha + y\sin\alpha - \sqrt{a^2\cos^2\alpha + b^2\sin^2\alpha} = 0$$

$x = c$

$y = 0$

$$\rho = \sqrt{a^2\cos^2\alpha + b^2\sin^2\alpha} - c\cos\alpha$$ [illegible]?

$$\rho^2 + 2c\rho\cos\alpha + c^2\cos^2\alpha = a^2\cos^2\alpha + b^2\sin^2\alpha$$

$$\rho^2 + 2c\rho\cos\alpha = a^2\cos^2\alpha + b^2\sin^2\alpha - c^2\cos^2\alpha$$

$$\rho^2 + 2c\rho\cos\alpha = b^2$$

~~$y = \rho$~~

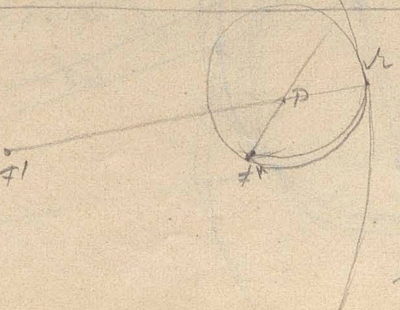
$$\cos\alpha = \frac{x-c}{\rho} \qquad \rho^2 = (x-c)^2 + y^2$$

$$\rho\cos\alpha = x - c$$

$$(x-c)^2 + y^2 + 2c(x-c) = b^2$$

$$x^2 + y^2 - 2cx + c^2 + 2cx - 2c^2 = b^2$$

$$x^2 + y^2 = b^2 + c^2 = a^2 = \frown$$



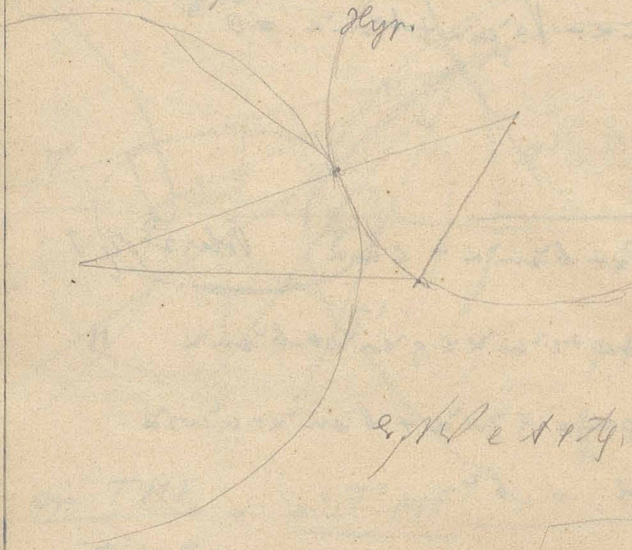
$PF + PF' = 2a$

$PM = PF$

$F'M = 2a$

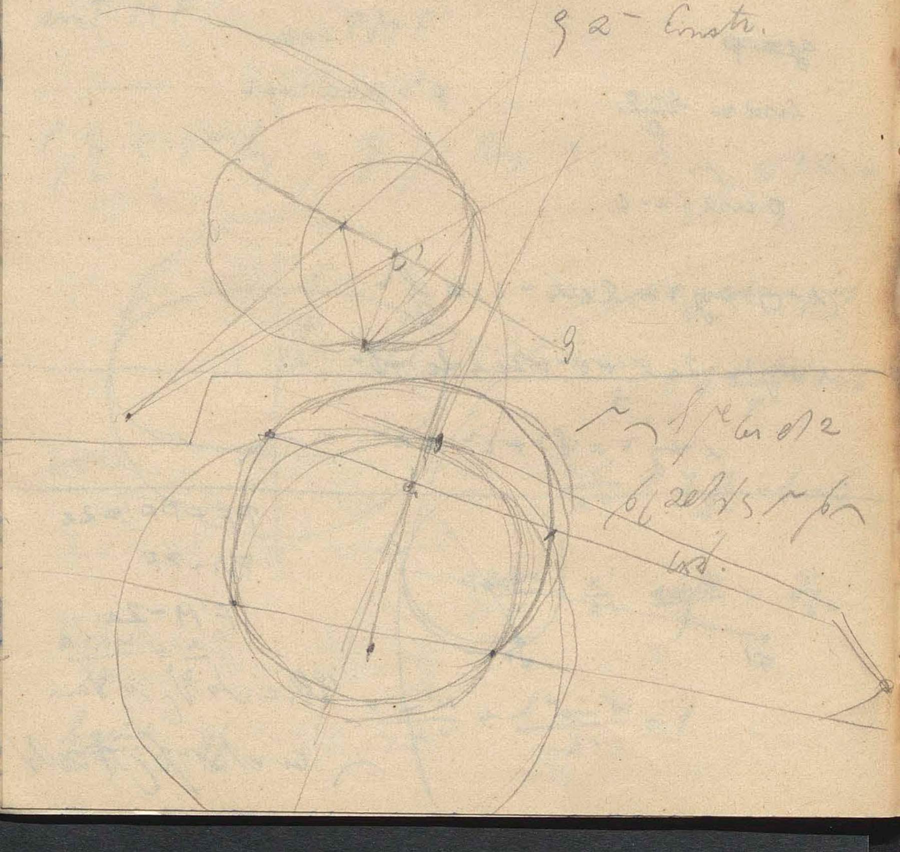
[illegible]

[illegible]



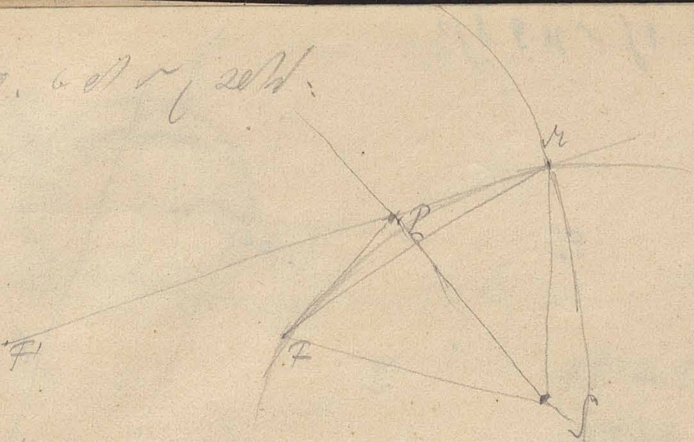
[illegible]

[illegible] Constr.



[illegible]

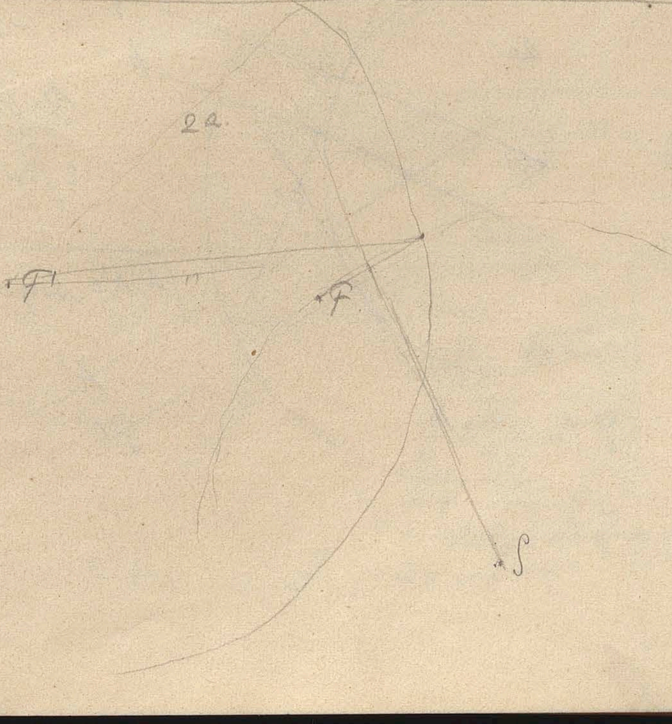
[illegible]



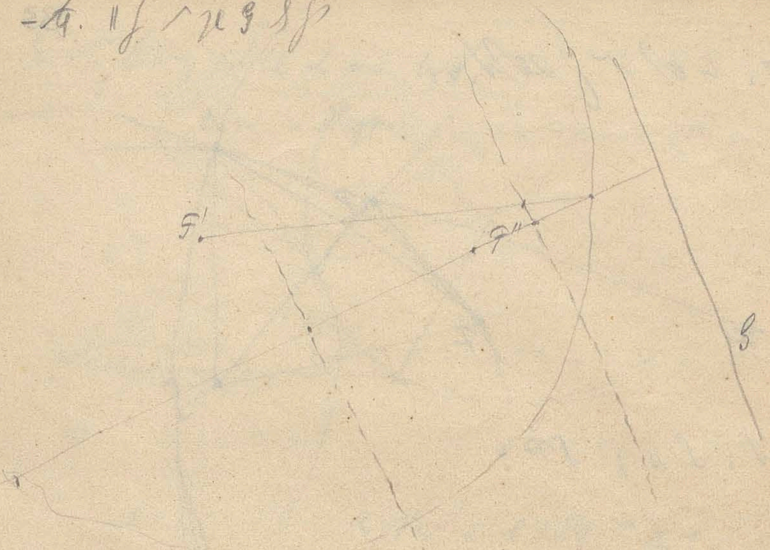
[illegible]: SM ; SP ?

$SM = SF$

$MF' = a$ } [illegible]

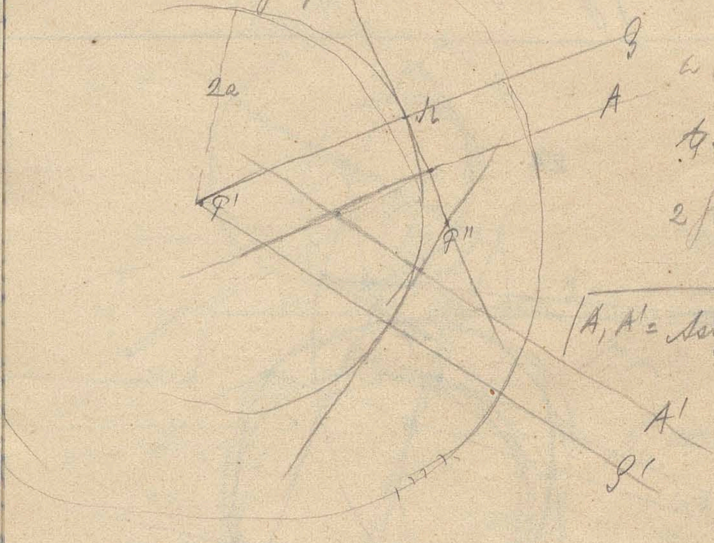


[illegible]



[illegible] Hyp. [illegible]

[illegible] Asympt.

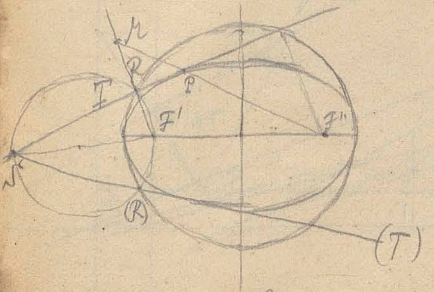


[illegible] g ‖

[illegible]

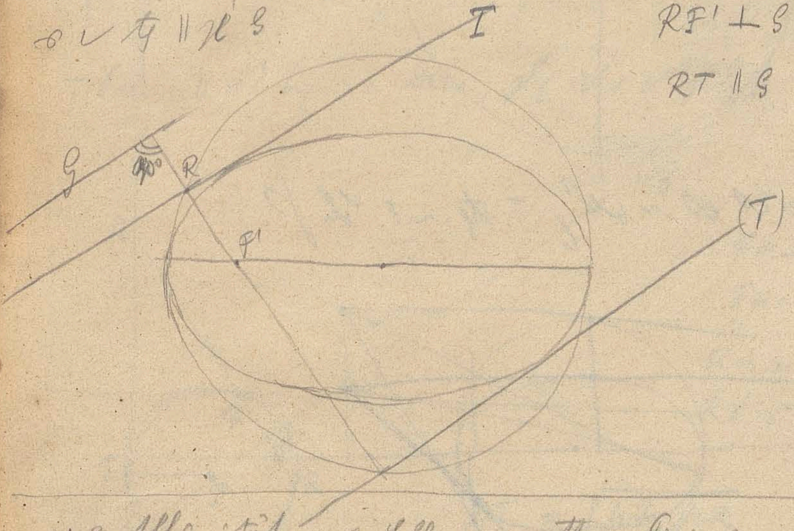
2 [illegible]

A, A' = Asympt.

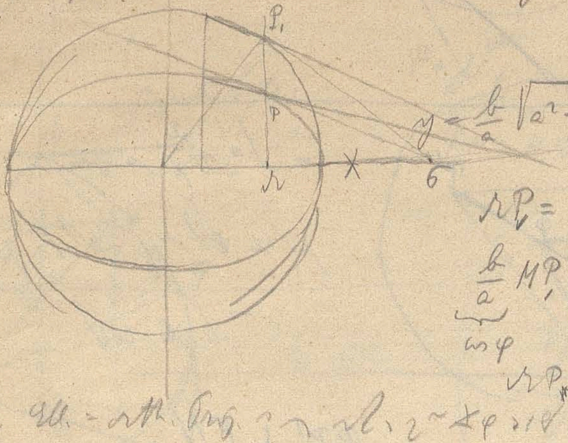


$RF' \perp g$

$RT \parallel g$



… Affinität: … Ell. … orthog. Proj. …



$y = \frac{b}{a}\sqrt{a^2-x^2}$

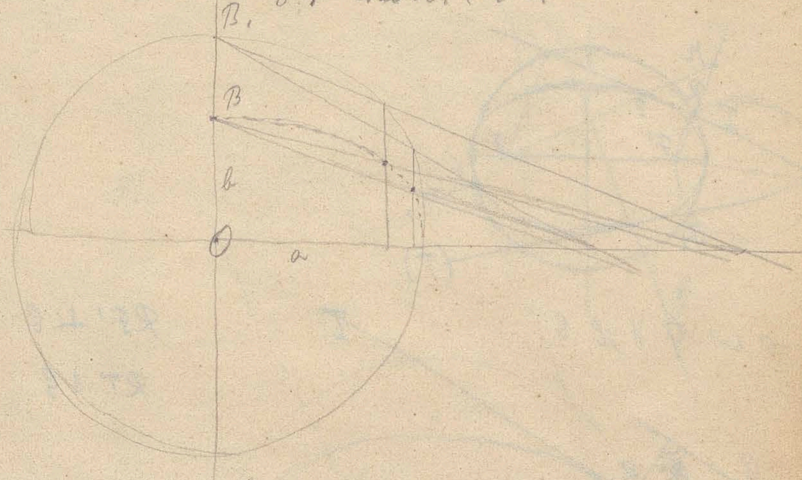
$MP_1 = \sqrt{a^2-x^2}$

$\underbrace{\frac{b}{a}}_{\cos\varphi} MP_1 = MP_I$

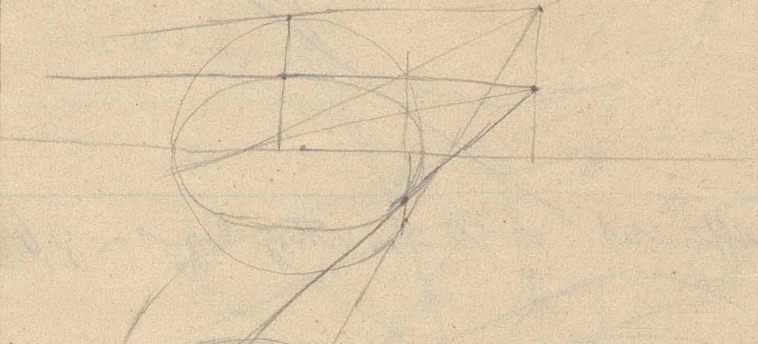
$MP_\pi = MP_1 \cdot \cos\varphi$

… Ell. … orth. Proj. … $\measuredangle\varphi$ …

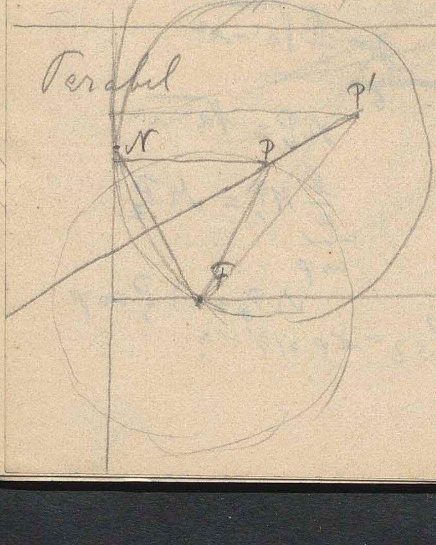
[illegible] Constr. d. Ell.



[illegible]



Parabel

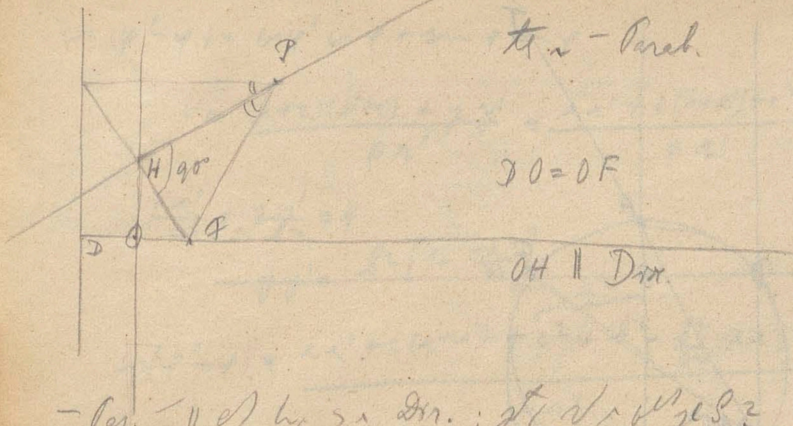


= [illegible]

[illegible]

[illegible]

$PP' \perp FN$

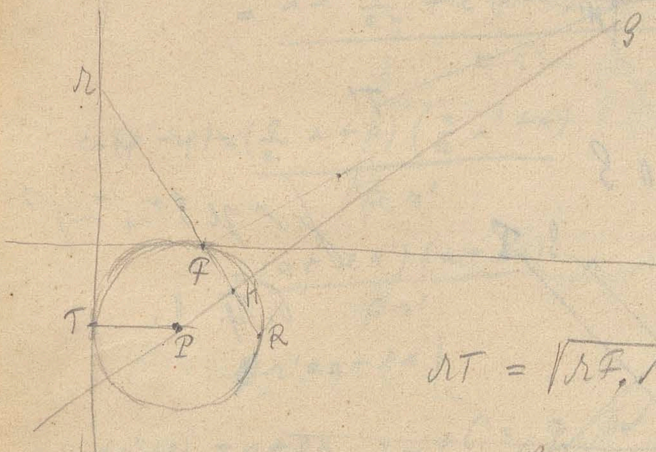


[illegible] – Parab.

$DO = OF$

$OH \parallel$ Dir.

– Par. [illegible] ∥ [illegible] Dir. ; [illegible] ?



$FH \perp g$

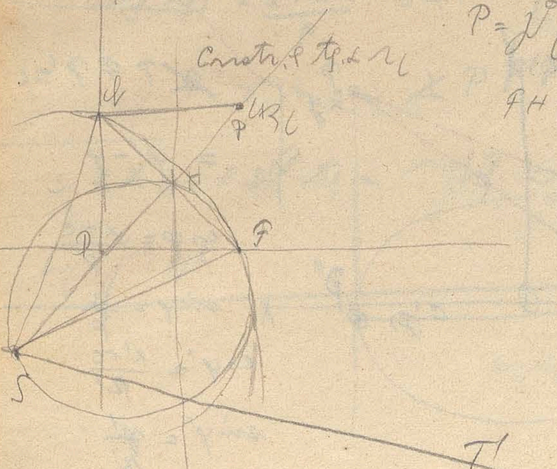
$FH = HR$

[illegible]

R [illegible] g

$rT = \sqrt{rF \cdot rR}$

Constr. [illegible]

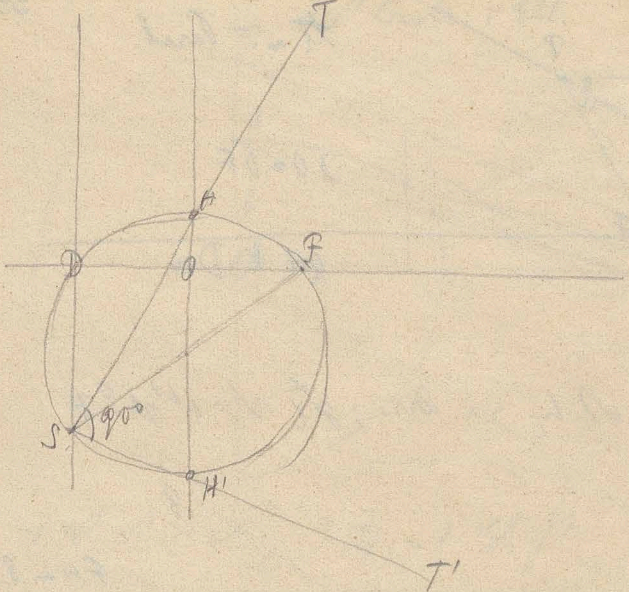


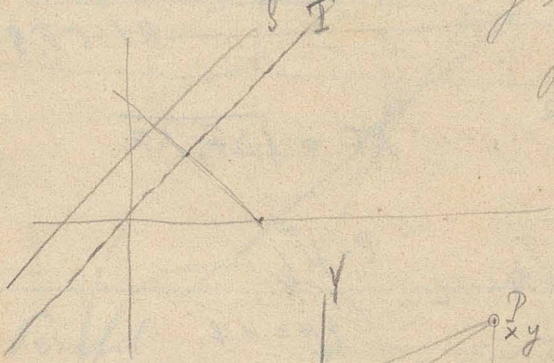
$P = [illegible]$

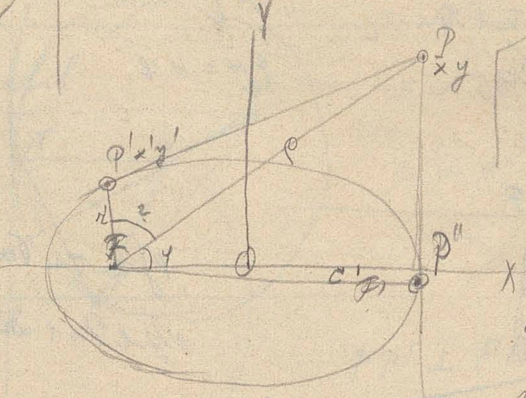
$FH = Hd$

[illegible] ⊥ Par. [illegible]

[illegible] Dir.







$\angle PFP' = \tau$

$= \varphi' - \varphi$

$\cos\varphi = \frac{x+c}{\rho}$

$\sin\varphi = \frac{y}{\rho}$

$\cos\varphi' = \frac{x'+c}{\rho'}$

$\sin\varphi' = \frac{y'}{\rho'}$

$$\cos(\varphi'-\varphi) = \cos\varphi'\cos\varphi + \sin\varphi'\sin\varphi$$

$$= \frac{(x+c)(x'+c) + yy'}{\rho r'} = \frac{xx' + c(x+x') + c^2 + yy'}{\rho r'}$$

$$\frac{xx'}{a^2} + \frac{yy'}{b^2} = 1$$

$$yy' = b^2\left[1 - \frac{xx'}{a^2}\right]$$

$$\cos(\varphi'-\varphi) = \frac{xx' + c(x+x') + \overbrace{c^2 + b^2}^{a^2} - \frac{b^2}{a^2}(xx')}{\rho r'}$$

$$= \frac{xx'\frac{c^2}{a^2} + c(x+x') + a^2}{\rho r'}$$

$$\cos(\varphi'-\varphi) = \frac{\left(\frac{c}{a}x + a\right)\left(\frac{c}{a}x' + a\right)}{\rho r'}$$

$$= \frac{(a+ex')(a+ex)}{\rho r'}$$

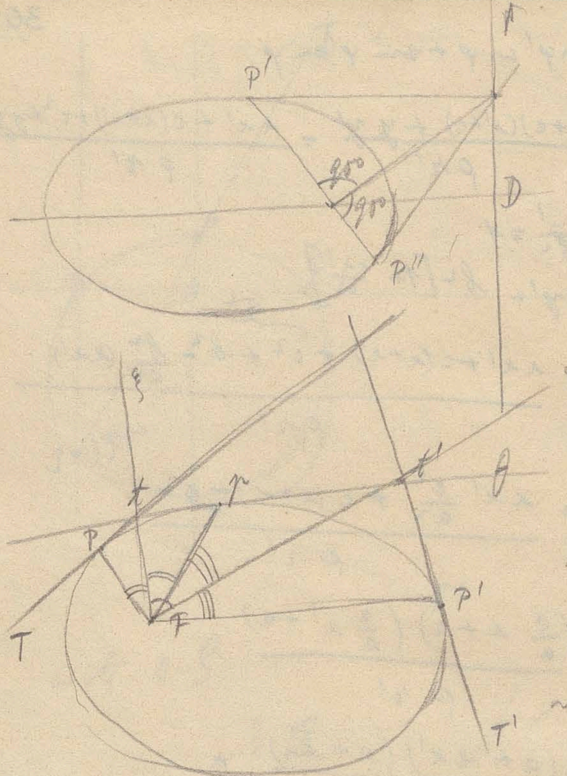
$$r' = a + ex'$$

$\cos(\varphi'-\varphi) = \frac{a+ex}{\rho}$ [illegible]

[illegible] P'' [illegible] $\measuredangle PF'P' = \measuredangle PF'P''$

[illegible]

[illegible]



[illegible] Dir.

[illegible] Involution

2 [illegible] TT'

[illegible] θ

[illegible]

[illegible] const. ∡ [illegible]

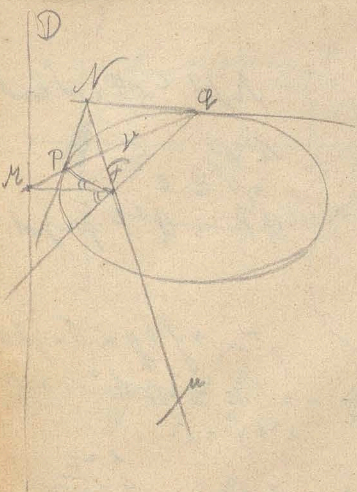
$\measuredangle\, tFt' = \frac{1}{2}\, PFP'$

[illegible] projekt. [illegible]

ξ ξ' [illegible] 2 [illegible] 2 concentr. [illegible]

[illegible] 2 [illegible] 4 [illegible]

[illegible] projekt. [illegible]



[illegible]

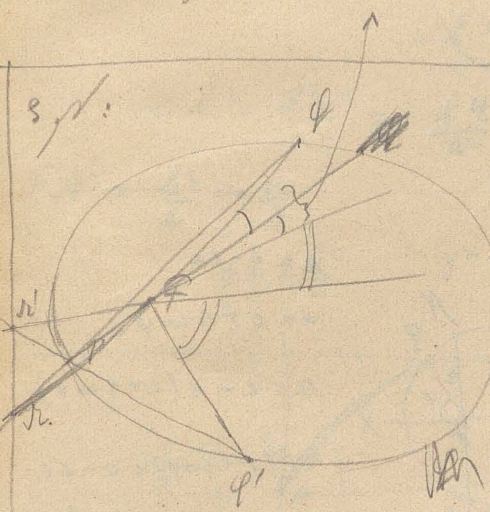
[illegible]

$\angle PFN = \angle NFQ$

μ, FM [illegible]

[illegible]

$FM \perp \mu$



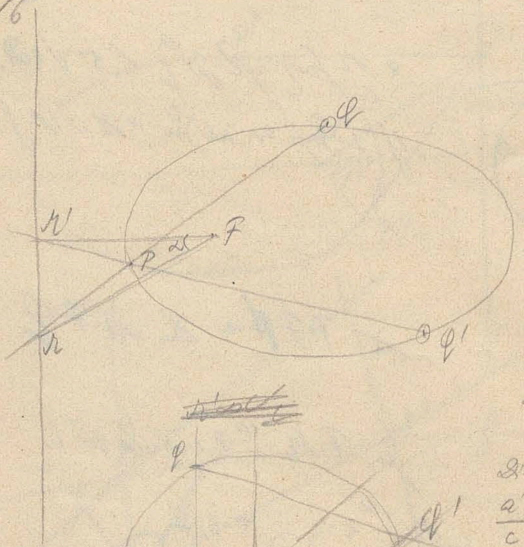
Q, Q' [illegible], P var.

[illegible]

[illegible] const. $\angle$ [illegible]

$$MFM' = \angle + \angle$$
$$= \frac{1}{2}\left[2\angle + 2\angle\right]$$
$$= \frac{1}{2}\, QFQ'$$

23/6



[illegible]

[illegible]

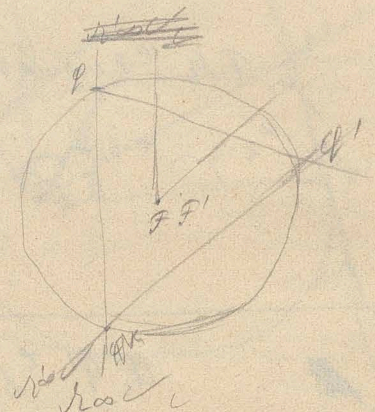
QP [illegible] $Q'P$ [illegible]

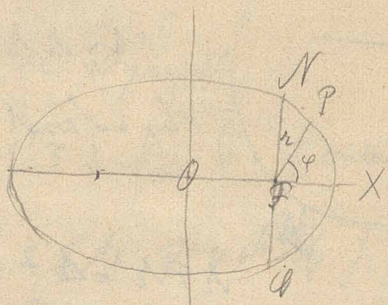
[illegible]

$x^2+y^2=a^2$ $e=0$

Directrix

$\frac{a^2}{c}=\infty$ $=\infty$ [illegible]





$r = a - ex$

$x = c + r\cos\varphi$

$r = a - e(c + r\cos\varphi)$

$r(1+e\cos\varphi) = a - ec$

$= a(1-e^2)$

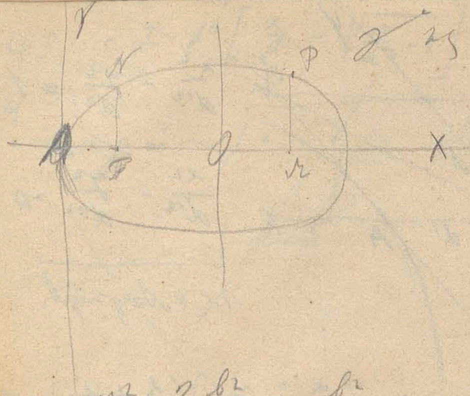
$r = \frac{a(1-e^2)}{1+e\cos\varphi}$

$NQ = p =$ Parameter

$\frac{p}{2} = a(1-e^2)$

$r = \frac{p}{2(1+e\cos\varphi)}$

[illegible]



(B). $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

(A). $\frac{(x-a)^2}{a^2}+\frac{y^2}{b^2}=1$

$$\frac{x^2}{a^2}-\frac{2x}{a}+\frac{y^2}{b^2}=0$$

$$\frac{y^2}{b^2}=2\frac{x}{a}-\frac{x^2}{a^2}$$

$$y^2 = 2\frac{b^2}{a}x-\frac{b^2}{a^2}x^2$$

$$\overline{FN}^2 = 2\frac{b^2}{a}(a-c)-\frac{b^2}{a^2}(a-c)^2$$

$$= 2b^2-\frac{2b^2c}{a}-\underbrace{\frac{b^2}{a^2}(a^2-2ac+c^2)}$$

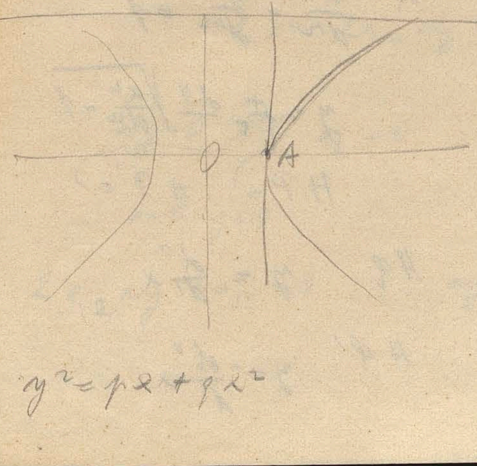
$$-b^2+\frac{2b^2c}{a}-\frac{b^2}{a^2}c^2$$

$$= b^2-\frac{b^2c^2}{a^2} = \frac{b^4}{a^2}$$

$$FN = \frac{b^2}{a} = \frac{p}{2}$$

$$p = 2\frac{b^2}{a} \qquad y^2 = px-\underbrace{\frac{b^2}{a^2}}_{q}x^2$$

$$= px - qx^2$$



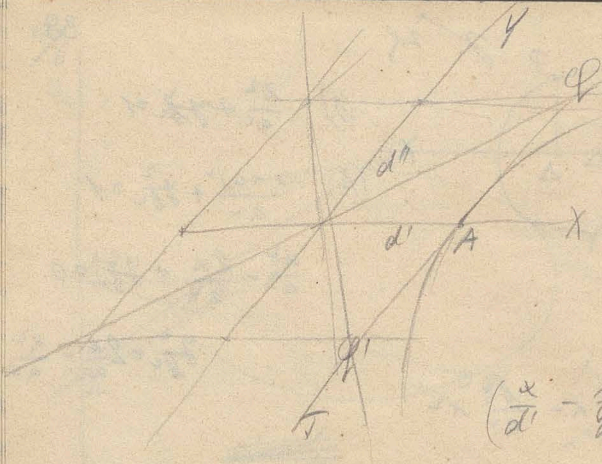
$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$\frac{(x+a)^2}{a^2}-\frac{y^2}{b^2}=1$$

$$\frac{x^2}{a^2}+\frac{2x}{a}+1-\frac{y^2}{b^2}=0$$

$$y^2=2\underbrace{\frac{b^2}{a}}_{p}x+\frac{b^2}{a^2}x^2.$$

$$y^2 = px + qx^2$$



$$\frac{x^2}{d'^2} - \frac{y^2}{d''^2} = 1$$

$$\frac{x^2}{d'^2} - \frac{y^2}{d''^2} = 0$$

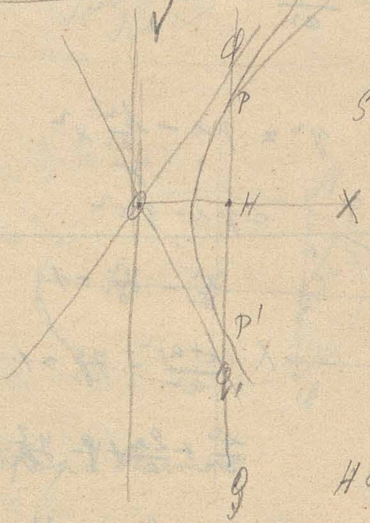
[illegible] Asympt.

$$\left(\frac{x}{d'} - \frac{y}{d''}\right)\left(\frac{x}{d'} + \frac{y}{d''}\right) = 0$$

$x = d'$ $y = 0$

~~[illegible]~~ [illegible] Hyperb. [illegible] 2 Asympt. [illegible]
[illegible]

[illegible] A = Q



$QP = Q'P'$

[illegible]
[illegible]

$$\frac{x^2}{d'^2} - \frac{y^2}{d''^2} = 1$$

$$y = \pm\, d'' \sqrt{\frac{x^2}{d'^2} - 1}$$

HP = HP' [illegible]

HQ $y = \frac{d''}{d'}x$

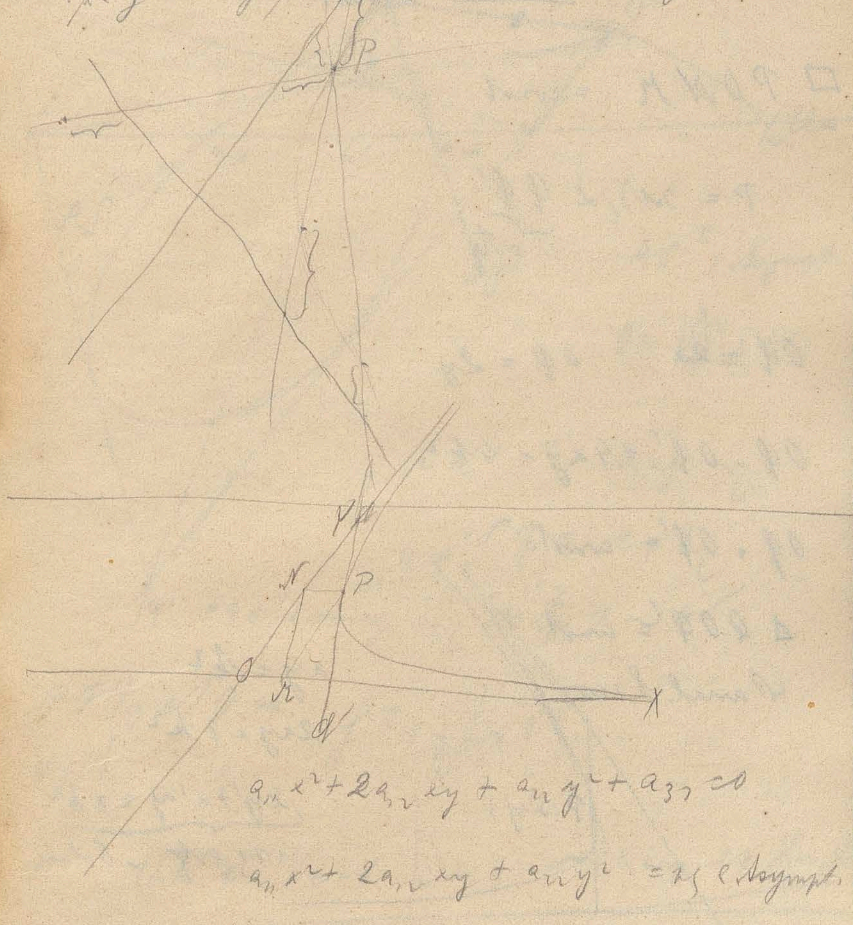
HQ' $y = -\frac{d''}{d'}x$

$\varphi P = \pm d'' \left[\frac{x}{d'} - \sqrt{\frac{x^2}{d'^2} - 1} \right]$

$d'P' = 1$

$\rho_{x^2} \int$ ~ $\int$ ...

Constr. Hyp.



$a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + a_{33} = 0$

$a_{11}x^2 + 2a_{12}xy + a_{22}y^2 = 0$ Asympt.

$xy = 0 \qquad a_{11} = 0 \qquad a_{22} = 0$

$x = 0,\ y = 0$

$2a_{12}xy + a_{33} = 0 \qquad xy = k^2$ Curve

$xy \sin\alpha = 2 \Delta NOM$

$\Delta NOM = \frac{k^2 \sin\alpha}{2} = const.$

$\square\ PONM = const$

$P = $ m) ($\measuredangle$ QQ' $= t_g$

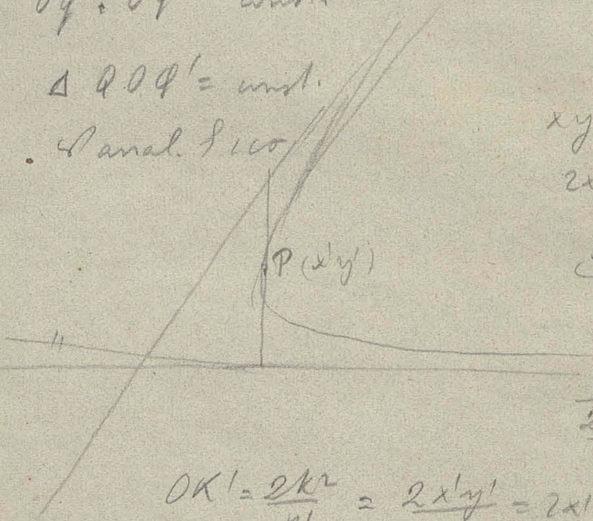
$OQ' = 2x \qquad OQ = 2y$

$OQ \cdot OQ' = 4xy = 4k^2$

$OQ \cdot OQ' = const.$

$\Delta\ QOQ' = const.$

anal. Seco



$xy = k^2$

$2xy = 2k^2$

$xy' + x'y = 2k^2$

rg e tg. Polare

$$\frac{x}{\frac{2k^2}{y'}} + \frac{y}{\frac{2k^2}{x'}} = 1$$

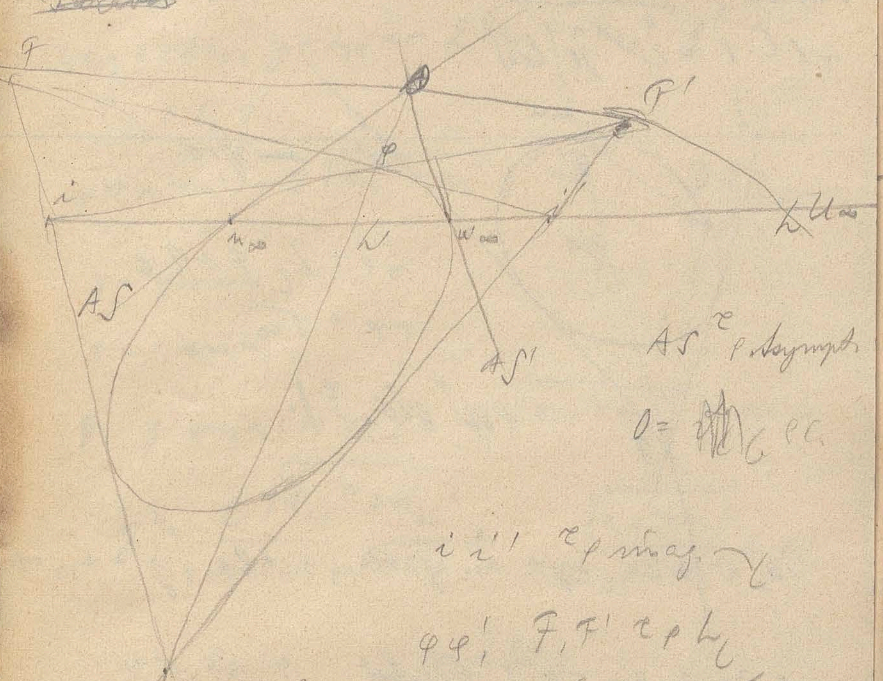
$$OK' = \frac{2k^2}{y'} = \frac{2x'y'}{y'} = 2x'$$

$$OQ = \frac{2k^2}{x'} = 2y'$$

$QOQ' = \frac{1}{2}\, OQ \cdot OQ' \sin\alpha = \frac{1}{2}\, OQ \quad = 2k^2 \sin\alpha$

$= const.$

~~Parabel~~



AS ... Asympt.

O = ...

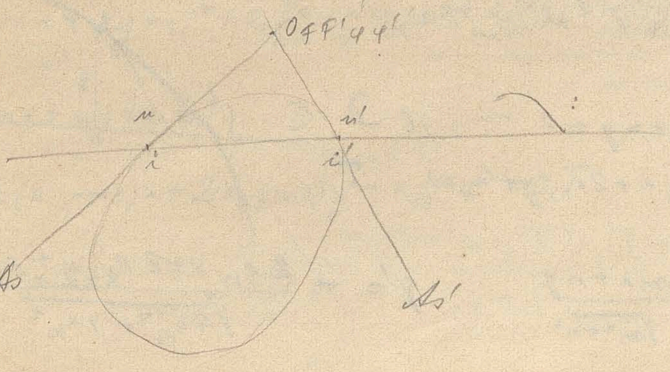
i i' ...

φ φ', F, F' ...

φ' ...

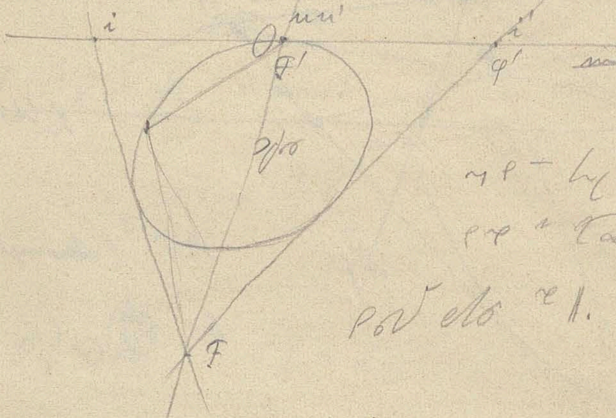
FF' / φφ' ... O ...

$(LL'\,i\,i') = -1$ | ...



Parabel

[illegible]



[illegible]

$$a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_{13}x + 2a_{23}y + a_{33} = 0$$

$$a_{12}^2 - a_{11}a_{22} = 0$$ } Parabel

$$a_{12} = \sqrt{a_{11}a_{22}}$$

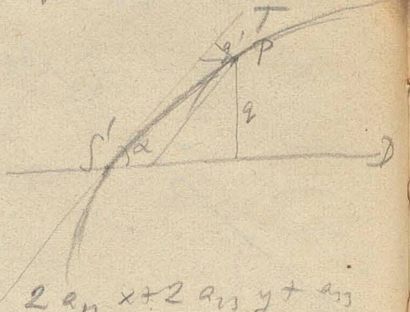
$$(x\sqrt{a_{11}} + y\sqrt{a_{22}})^2 + 2a_{13}x + 2a_{23}y + a_{33} = 0$$

$$(mx+ny)^2 + 2a_{13}x + 2a_{23}y + a_{33} = 0$$

[illegible]

$mx+ny=0$ — [illegible] D

$2a_{13}x+2a_{23}y+a_{33}=0$ T



$$q = \frac{mx+ny}{\sqrt{m^2+n^2}} \qquad q' = -\frac{2a_{13}x+2a_{23}y+a_{33}}{\sqrt{4a_{13}^2+4a_{23}^2}}$$

$mx + ny = q\sqrt{m^2+n^2}$

$2a_{13}x + 2a_{23}y + a_{33} = -2\sqrt{a_{13}^2 + a_{23}^2}\, q'$

ω P g e C:

$q^2(m^2+n^2) - 2\sqrt{a_{13}^2+a_{23}^2}\, q' = 0$ D X, T Y } ro

$q = y \sin\alpha$

$q' = x \sin\alpha$

$y^2 \sin^2\alpha\,(m^2+n^2) = 2\sqrt{a_{13}^2+a_{23}^2}\, x \sin\alpha$

$y^2 = \dfrac{2\sqrt{a_{13}^2+a_{23}^2}}{\sin\alpha\,(m^2+n^2)}\, x$ $\dfrac{\sqrt{a_{13}^2+a_{13}^2}}{\sin\alpha\,(m^2+n^2)} = p'$

$y^2 = 2p'x$ ∫ u x 2y

$y = \pm\sqrt{}$ D = elo e C

6/7 $(mx+ny+c)^2 + 2a_{13}x + 2a_{13}y + a_{33} = 0$

$-2mcx - 2ncy - c^2$

$(mx+ny+c)^2 + 2(a_{13}-mc)x + 2(a_{13}-nc)y + a_{33} - c^2 = 0$

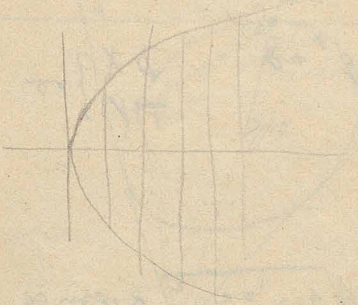
$mx+ny+c=0$ D

$2(a_{13}-mc)x + 2(a_{23}-nc)y + a_{33} - c^2 = 0$ T

$y^2 = 2px$ a elo ∥

[illegible]

$D = 0$



$Ax + By + C = 0 \mid A'x + \ldots$

$AA' + BB' = 0$ [illegible] $\perp$

$m(\quad) + n(a \quad) = 0$

$$C = \frac{m a_{13} + n a_{23}}{m^2 + n^2}$$

Symmetrieaxe

$$mx + ny + \frac{m a_{13} + n a_{23}}{m^2 + n^2} = 0$$ [illegible]

$$[illegible]\ 2\left(a_{13} - \frac{m^2 a_{13} + n m a_{23}}{m^2 + n^2}\right) + 2\left(a_{23} - \frac{n m a_{13} + n^2 a_{23}}{m^2 + n^2}\right) y + a_{33} - \left(\frac{m a_{13} + n a_{23}}{m^2 + n^2}\right)^2 = 0$$

$$2n \frac{(n a_{13} - m a_{23})}{m^2 + n^2} x - 2 \frac{m (n a_{13} - m a_{23})}{m^2 + n^2} y + a_{33} - (m a_{13} + n a_{23})^2 (m^2 + n^2)^2 = 0$$

$$(nx - my) + \frac{a_{33}(m^2 + n^2)}{2(n a_{13} - m a_{23})} - \frac{(m a_{13} + n a_{23})^2}{2(m^2 + n^2)(n a_{13} - m a_{23})} = 0$$

[illegible]

[illegible]

$$\frac{mx+ny+c}{\sqrt{m^2+n^2}}=0$$

T $$\frac{2(a_{13}-mc)x+2(a_{23}-nc)y+a_{33}-c^2}{2\sqrt{(a_{13}-mc)^2+(a_{23}-nc)^2}}=0$$

$$\frac{mx+ny+c}{\sqrt{m^2+n^2}}=\eta$$

↑

D = X [illegible]

T [illegible]

~~$2a_{13}-ncx$~~

$$\frac{2(a_{13}-mc)x+2(a_{23}-nc)y+a_{33}-c^2}{2\sqrt{(a_{13}-mc)^2+(a_{23}-nc)^2}}=-\xi$$

[illegible]:

$$\eta^2(m^2+n^2)=2\sqrt{(a_{13}-mc)^2+(a_{23}-nc)^2}\;\xi$$

$$\eta^2=2\,\frac{\sqrt{(a_{13}-mc)^2+(a_{23}-nc)^2}}{m^2+n^2}\;\xi$$

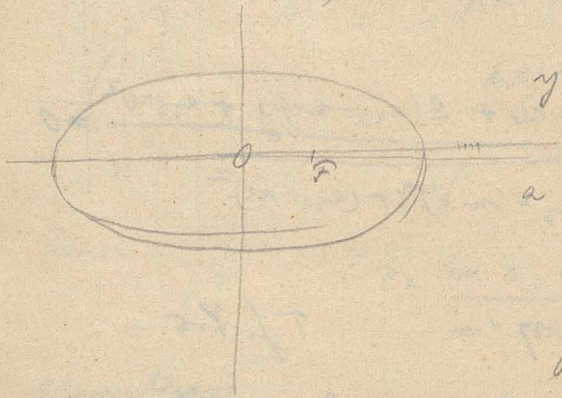
Hauptparam.

$$p=\frac{\sqrt{(a_{13}-mc)^2+(a_{23}-nc)^2}}{m^2+n^2}$$

$$p=\frac{\sqrt{\frac{n^2(na_{13}-ma_{23})^2}{(m^2+n^2)^2}+\frac{m^2(na_{13}-ma_{23})^2}{(m^2+n^2)^2}}}{m^2+n^2}$$

$p = \pm \frac{n a_{13} - m a_{23}}{(m^2+n^2)^{\frac{3}{2}}}$

Par. [illegible] Ell. [illegible] Hyp.



$y^2 = 2\frac{b^2}{a}x - \frac{b^2}{a^2}x^2$

a [illegible] ∞ [illegible]

[illegible]

$OF = \text{const.} = \frac{p}{2}$

$\frac{p}{2} = a - c \qquad c = a - \frac{p}{2}$

~~$p^2 = a^2$~~

$y^2 = \left(\frac{2ap - \frac{p^2}{2}}{a}\right) x - \left(\frac{p}{a} - \frac{p^2}{4a^2}\right) x^2$

$y^2 = \left(2p - \frac{p^2}{2a}\right) x - \left(\frac{p}{a} - \frac{p^2}{4a^2}\right) x^2$

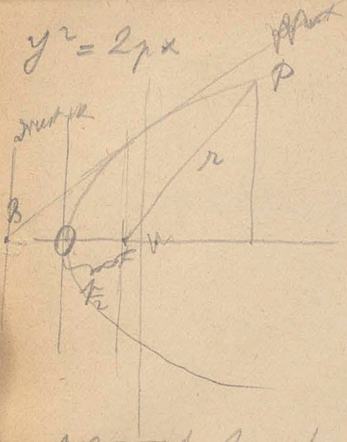
lim $a = \infty$

$y^2 = 2px$

[illegible] Hyperbel

$e = \frac{c}{a} = 1 - \frac{p}{2a}$ lim $a = \infty$, $e = 1$ } [illegible] Par. = Ell. / Hyp. } Exc = [illegible]

$y^2 = 2px$



$$r^2 = y^2 + \left(x - \frac{p}{2}\right)^2$$
$$= 2px + x^2 - px + \frac{p^2}{4}$$
$$= x^2 + px + \frac{p^2}{4} = \left(x + \frac{p}{2}\right)^2$$

$$r = x + \frac{p}{2}$$

Eq. P tg.

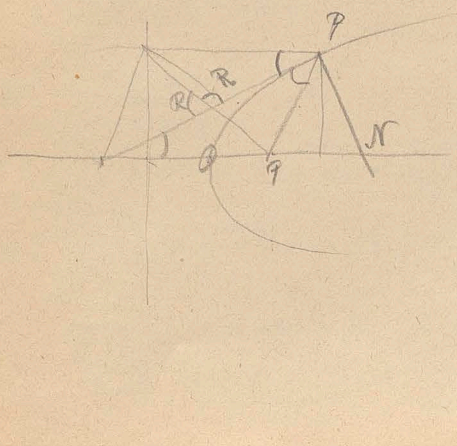
Eq. e tg. : ~~$y y' = p(x + x')$~~

$$y\eta = p(x + \xi)$$

$\eta = 0$ $\quad 0 = p(x + \xi)$

$\xi = -x$ $\qquad OT = -x$

Constr. e Par. tg.



$FT = TO + OF$

$= x + \frac{p}{2}$

$FT = FP$ [shorthand]

φT

$y\eta = p(x + \xi)$

$\frac{p}{y} = \frac{\eta}{\xi}$ const.

$\eta - y = -\frac{y}{p}(\xi - x)$ [shorthand] Norm

$\eta = 0 \mid \quad -y = \frac{y}{p}(\xi - x)$

$p = \xi - x$

$\xi = x + p$

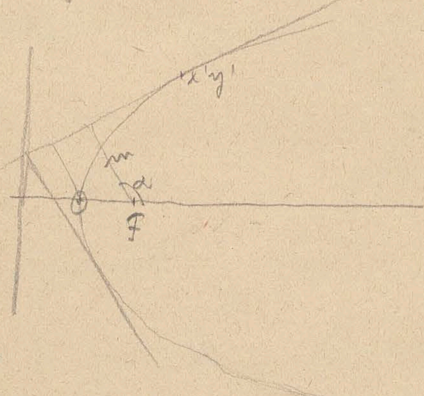
$ON = x + p$

$ON = x + MN$

$MN = p$

Subnorm. $= p$

Subtg. $= 2x$



$x'y' = p(x + x')$

$px - p'y + px = 0$

$\frac{px - x'y +}{-\sqrt{y^2 + y'^2}} = 0$

$\cos\alpha = -\frac{p}{\sqrt{p^2 + y'^2}}$

$\sin\alpha = \frac{y'}{\sqrt{p^2 + y'^2}}$

$$\frac{px - y'y + pe'}{\sqrt{p^2+y'^2}} = \text{[illegible]}\ (x'y')$$

$m = \left| \begin{array}{l} y=0 \\ x=\frac{p}{2} \end{array} \right.$ $\qquad m = \dfrac{p\frac{p}{2} + pe'}{\sqrt{p^2+y'^2}}$

$$= \frac{p^2+2pe'}{2\sqrt{p^2+y'^2}} = \frac{p^2+y'^2}{2\sqrt{p^2+y'^2}} =$$

$$= \frac{\sqrt{p^2+y'^2}}{2}$$

$$m = \frac{1}{2}\left(-\frac{p}{\cos\alpha}\right) = -\frac{p}{2\cos\alpha}$$

[illegible]: $x\cos\alpha + y\sin\alpha - m = 0$

$$x\cos\alpha + y\sin\alpha + \frac{p}{2\cos\alpha} = 0$$

$$x\cos^2\alpha + y\cos\alpha\sin\alpha + \frac{p}{2} = 0$$ | [illegible] [illegible] α | [illegible]

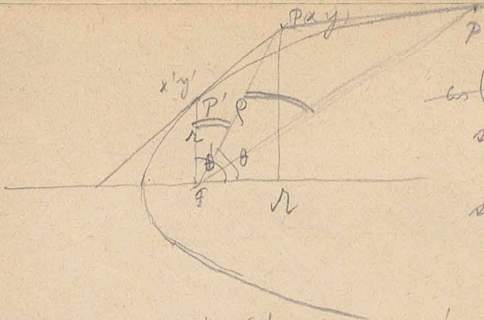
[illegible] D.

α — $\alpha + 90$ $\quad T'$

T' — $x\sin^2\alpha - y\sin\alpha\cos\alpha + \frac{p}{2} = 0$

$T + T'$ $\quad x + p = 0$ = [illegible]

$x = -p$ [illegible]



~~$\cos(\theta' - \theta) =$~~

$\sin\theta = y/\rho \quad \cos\theta = \frac{x - p/2}{\rho}$

$\sin\theta' = y'/r \quad \cos\theta' = \frac{x' - p/2}{r}$

$$\cos(\theta' - \theta) = \frac{(x - p/2)(x' - p/2) + yy'}{r\rho} = \frac{xx' - \frac{p}{2}(x + x') + \frac{p^2}{4} + p(x+x')}{r\rho}$$

$$= \frac{xx' + \frac{p}{2}(x + x') + \frac{p^2}{4}}{r\rho} = \frac{(x + p/2)(x' + p/2)}{r\rho}$$

$r = x' + p/2$ $\qquad \cos(\theta' - \theta) = \frac{x + p/2}{\rho}$

[illegible] $\angle P'$ [illegible] $= \angle P''$

[illegible] Ell. u. Hyp.

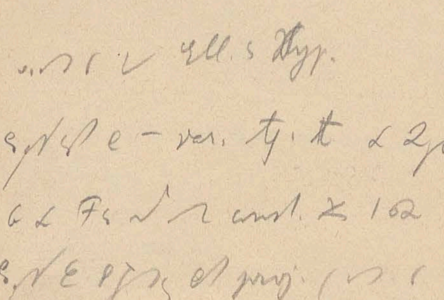
[illegible] T, T' [illegible]

[illegible] F [illegible] const. [illegible]

[illegible] H.

[illegible] P [illegible]

[illegible] per. $p' = ?$



$y^2 = 2px$

[illegible] x, y.

$x = \xi + x'$

$(\eta + y_1)^2 = 2p(\xi + x_1)$

$\eta^2 + 2\eta y_1 + y_1^2 = 2p\xi + 2px_1$

$2\eta y_1 + y_1^2 = 2px_1$

$y_1 = y' \sin\omega$

$x_1 = x' + y' \cos\omega$

$2\eta y' \sin\omega + y'^2 \sin^2\omega = 2px' + 2py' \cos\omega$

$y^2 = 2px'$

$2\eta y' \sin\omega = 2py' \cos\omega$

$$\operatorname{tg}\omega = \frac{p}{\eta}$$

$$y'^2 \sin^2\omega = 2px'$$

$$y'^2 = \frac{2p}{\sin^2\omega} x'$$

$$p' = \frac{p}{\sin^2\omega}$$

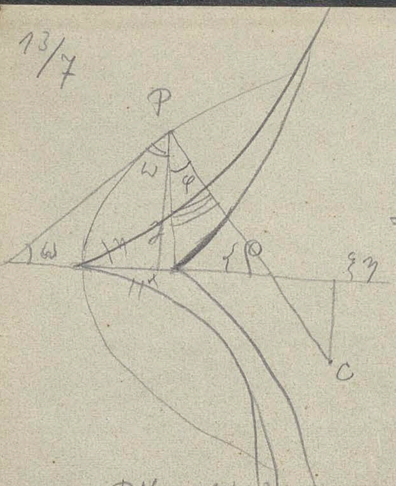
$$\sin^2\omega = \frac{\operatorname{tg}^2\omega}{1+\operatorname{tg}^2\omega} = \frac{\frac{p^2}{\eta^2}}{1+\frac{p^2}{\eta^2}} = \frac{p^2}{\eta^2+p^2}$$

$$p' = \frac{p(\eta^2+p^2)}{p^2} = \frac{\eta^2+p^2}{p} = p + 2\xi$$

$TN = p'$

Opherzy e Par. !!!

13/7



$$PC = \rho = \frac{PN}{\cos^2\varphi} = \frac{PN}{\sin^2\omega}$$

$$\operatorname{tg}\omega = \frac{p}{y}$$

$$\sin^2\omega = \frac{\operatorname{tg}^2\omega}{1+\operatorname{tg}^2\omega} = \frac{\frac{p^2}{y^2}}{1+\frac{p^2}{y^2}} = \frac{p^2}{y^2+p^2}$$

$$\rho = PN\,\frac{y^2+p^2}{p^2}$$

$$\eta = y - \rho\cos\psi = y - \rho\frac{y}{PN}$$

$$= y\left[1-\frac{\rho}{PN}\right] = y\left[1-\frac{y^2+p^2}{p^2}\right] = \frac{-y^3}{p^2}$$

$$\xi = x + \rho\cos\psi = x + \rho\frac{MN}{PN} = x + \frac{\rho}{PN}p$$

$$= x + \frac{y^2+p^2}{p^2}p = x + \frac{2px+p^2}{p} = 3x + p$$

$$y = -\sqrt[3]{p^2\eta} \qquad x = \frac{\xi-p}{3}$$

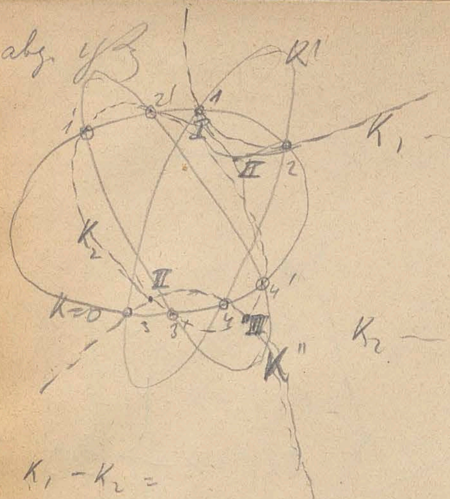
$$y^2 = 2px \qquad \sqrt[3]{p^4\eta^2} = 2p\left(\frac{\xi-p}{3}\right)$$

Evolute d. Par. = Neil'sche Par.

$$p^4\eta^2 = 8p^3\frac{(\xi-p)^3}{27}$$

$$\eta^2 = 8\frac{(\xi-p)^3}{27p}$$

abg. [illegible]



$K_1 \sim K - \lambda K' = 0$

= [illegible] 1234 [illegible]

$K_2 \sim K - \mu K'' = 0$

= [illegible] 1'2'3'4'

$K_1 - K_2 =$

$\lambda K' - \mu K'' = 0 \;\Big|\; K' - \frac{\mu}{\lambda} K'' = 0$

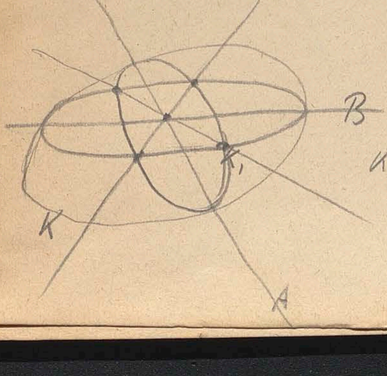
$K' - \nu K'' = 0$

[illegible] K_1 [illegible] K_2 [illegible] : I, II, III, IV,

[illegible] K' [illegible] K'' I', II', III', IV' [illegible]

[illegible] $K\, K'\, K''$ 3 [illegible] 2 [illegible] Quadr. [illegible] 2 [illegible] $K_1\, K_2$ [illegible] 4 [illegible] $K' K''$ [illegible]

Sätze v. Brianchon & Pascal



$K_1 \quad K + A^2 = 0$

$K_2 \quad K + B^2 = 0$

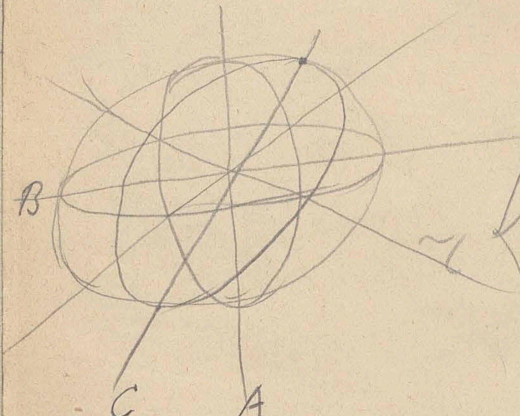
$K_1 - K_2 = 0 \quad A^2 - B^2 = 0$

$(A - B)(A + B) = 0$

[illegible] $\begin{matrix} A - B = 0 \\ A + B = 0 \end{matrix}$ [illegible (AB) illegible]

[illegible] $K_1 K_2$ [illegible] 3 K [illegible]

A, B [illegible] $K_1 K_2$ [illegible]



$A - B = 0$

$A + B = 0$

$B - C = 0$

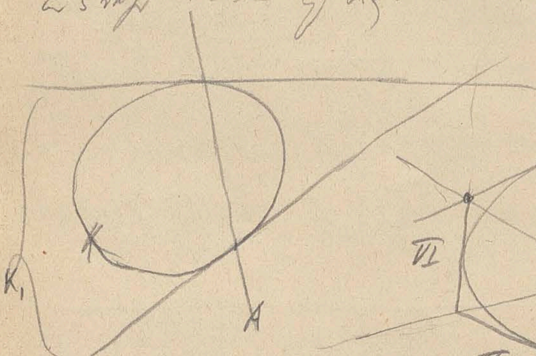
$B + C = 0$

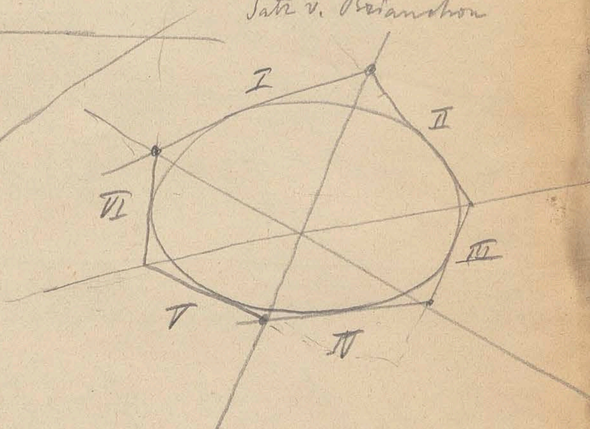
$C - A = 0$

$C + A = 0$

[illegible] 3 [illegible]

Satz v. Brianchon



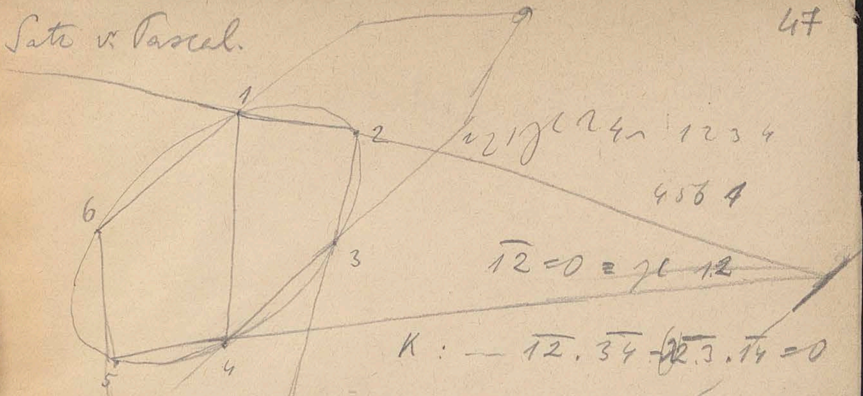


I, IV [illegible] K_1

II, V [illegible] K_2

III, VI [illegible] K_3

Satz v. Pascal.



[illegible] 1 2 3 4

4 5 6 1

$\overline{12} = 0$ = [illegible] 12

$K: \ldots \overline{12} \cdot \overline{34} - \overline{23} \cdot \overline{14} = 0$

$K: \ldots \overline{45} \cdot \overline{61} - \overline{56}\ \overline{14} = 0$

$\overline{12} \cdot \overline{34} - \overline{23} \cdot \overline{14} = \overline{45} \cdot \overline{61} - \overline{56} \cdot \overline{14}$

$\overline{12} \cdot \overline{34} - \overline{45} \cdot \overline{61} = (\overline{23} - \overline{56})\, \overline{14} = 0$

[illegible] 12 34 [illegible] 45 61 [illegible]

$\overline{23} - \overline{56} = 0$

$14 = 0$

Pascal-Linie